- These slides/notes represent only part of the course, and were accompanied by face-to-face explanations on white-board and additional topics / learning materials.
- In preparation of these slides I have also benefited from various books and online material.
- Some of the slides contain animations which may not be visible in pdf version.
- Corrections, comments, feedback may be sent to https://www.linkedin.com/in/naveedrazzaqbutt/

EE 202
Electric Circuit Analysis

with

Dr. Naveed R. Butt

@

Jouf University

Introductions …

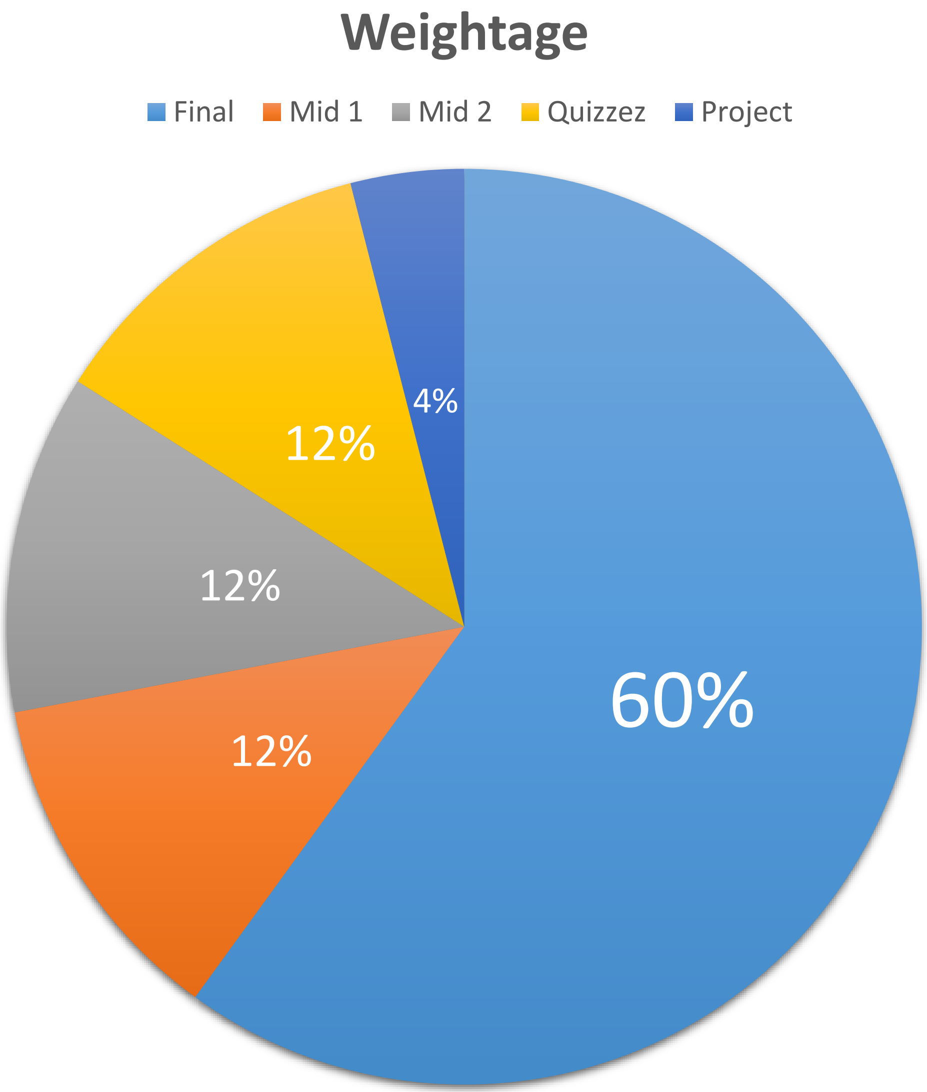
- Me
- You
- The Course

Important Business!!

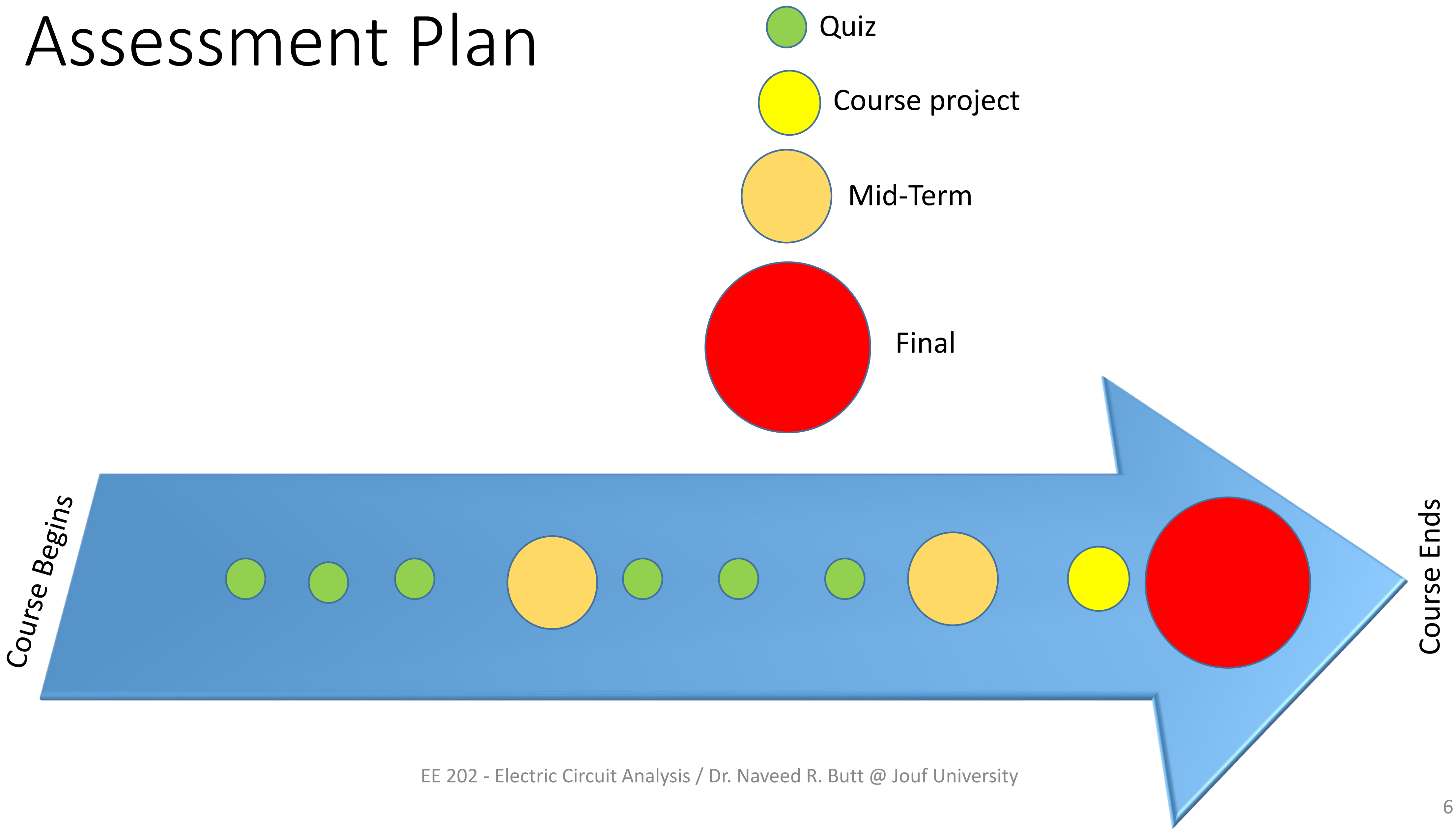
- 75% attendance is mandatory!
- Textbooks & Notes
  - Electric Circuits, J. Nilsson and S. Riedel, 2014
  - Lecture notes are posted on Blackboard https://lms.ju.edu.sa/
- Contact
  - nbutt@ju.edu.sa
  - office: 1021

Learning Plan

- **Lectures**
  - Help discover and grasp new concepts
- **Quizzes (six)**
  - Help prepare/revise each week's concepts
  - Keep you from lagging behind in course
- **Presentation**
  - Helps learn independent work & presentation
  - Prepares for final year project
- **Exams (Mid-1, Mid-2, Final)**
  - Help prepare entire course material



Assessment Plan



In this course we will discuss …

- Three-phase circuits and power calculations
- Linear op-amp and op-amp circuits
- Transient and steady-state response of the first-order and the second-order circuits
- Laplace transform and solution of circuits in complex-frequency domain
- Frequency response of passive circuits, transfer functions, poles and zeros,
- Resonance networks, and filters
- Two-Port networks
- Mutually-coupled coils and the ideal transformer

Course Learning Objectives (CLOs)

CLO #	Domain	Description	Assessment
CLO 1	Cognitive Skills	**Calculate** power factor corrections for basic electric circuits	HW, Quiz, Mid, Final
CLO 2	Cognitive Skills	**Calculate** parameters related to balanced three phase circuits	HW, Quiz, Mid, Final
CLO 3	Cognitive Skills	**Calculate** parameters related to transient behavior of first order circuit, and Laplace Transform.	HW, Quiz, Mid, Final
CLO 4	Cognitive Skills	**Analyze** the operational amplifiers and two port networks	HW, Final
CLO 5	Communication	**Demonstrate** the ability to research a topic related to electric circuits and formally present the results	Project Presentation

Questions?? Thoughts??

EE 202
Electric Circuit Analysis

with

Dr. Naveed R. Butt

@

Jouf University

Electric? Circuit? Analysis?

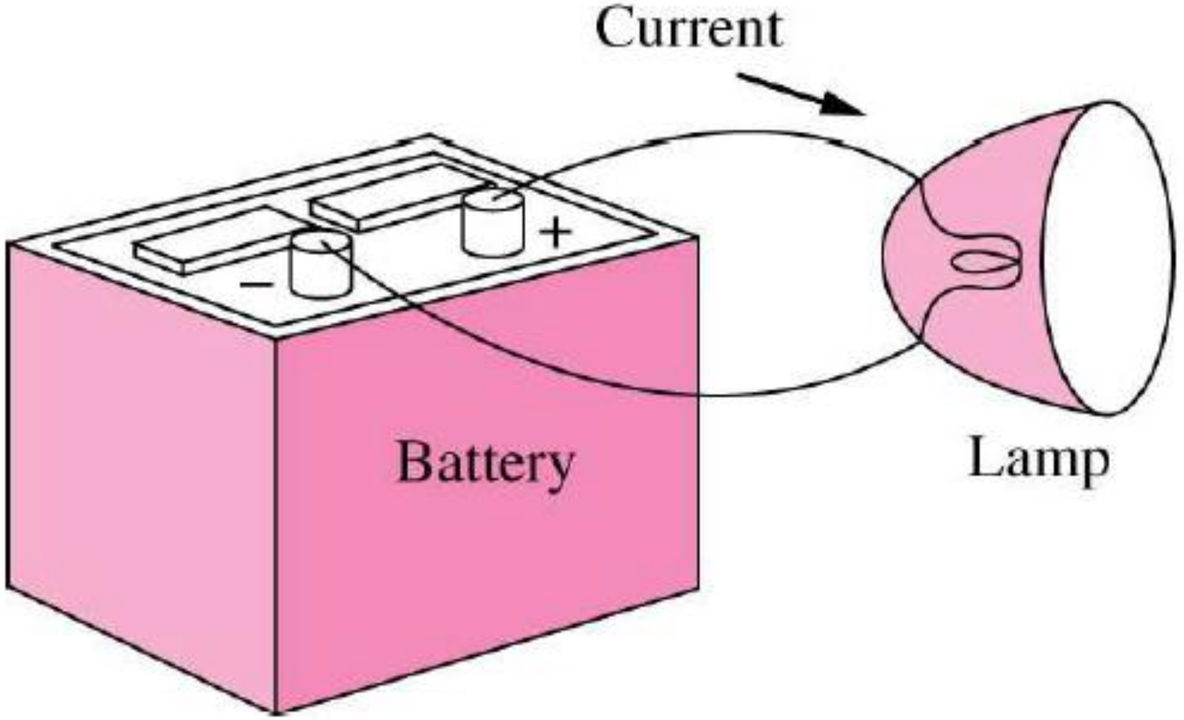
- What are each of these three!
- Electric?
  - What are the various fields? (electric, magnetic, gravitational…)
  - Where does the electric field exist?
  - Why is it so important to us now? (hint: electricity)
- Circuit?
  - Latin *circumire* "go around," from *circum* "round" + *ire* "to go“
  - Also recall “circle”
  - Electric Circuit (electricity going around)

Electric? Circuit? Analysis?

- What are each of these three!
- Analysis
  - What does “analysis” mean?
    - "resolution of anything complex into simple elements“
    - *ana* "throughout" + *lysis* "a loosening"
  - Why do we do it? (hints: understand, design, utilize, plan, avoid)

Circuits Everywhere

A Simple Circuit



Units and Prefixes

- SI Base Units
- SI prefixes for large and small quantities (“metric” system)

System of Units

The International System of Units, or Système International des Unités (**SI)**, also known as **metric** system uses 7 mutually independent base units. All other units are *derived* units.

Base quantity	SI base unit: Name	SI base unit: Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

SI Base Units

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

Current and Voltage

- Let’s talk about each

Electric Current (Charges in Motion!)

- **Current:** net flow of charge across any cross section of a conductor, measured in Amperes (Andre-Marie Ampere (1775-1836), a French mathematician and physicist)

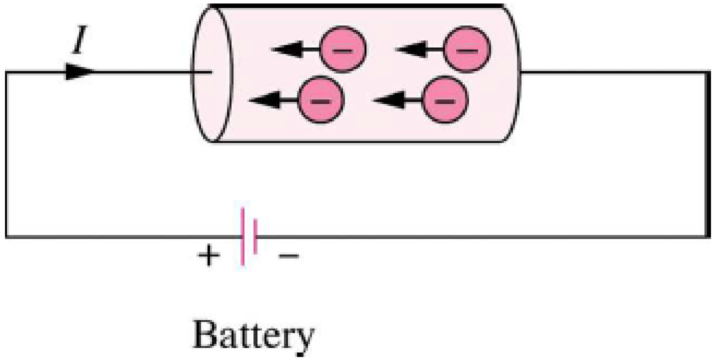
- Current can be thought of as the rate of change of charge:

$$i = \frac{dq}{dt}$$

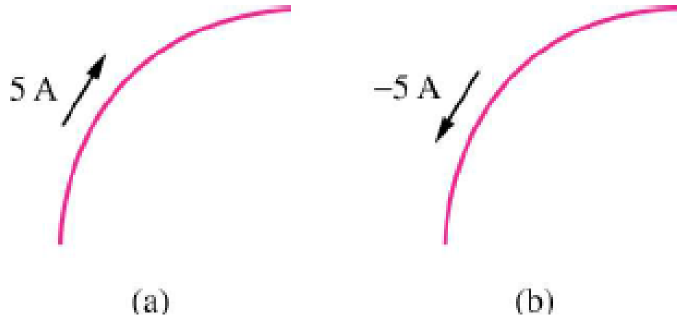
i = the current in amperes,
q = the charge in coulombs,
t = the time in seconds.

Electric Current

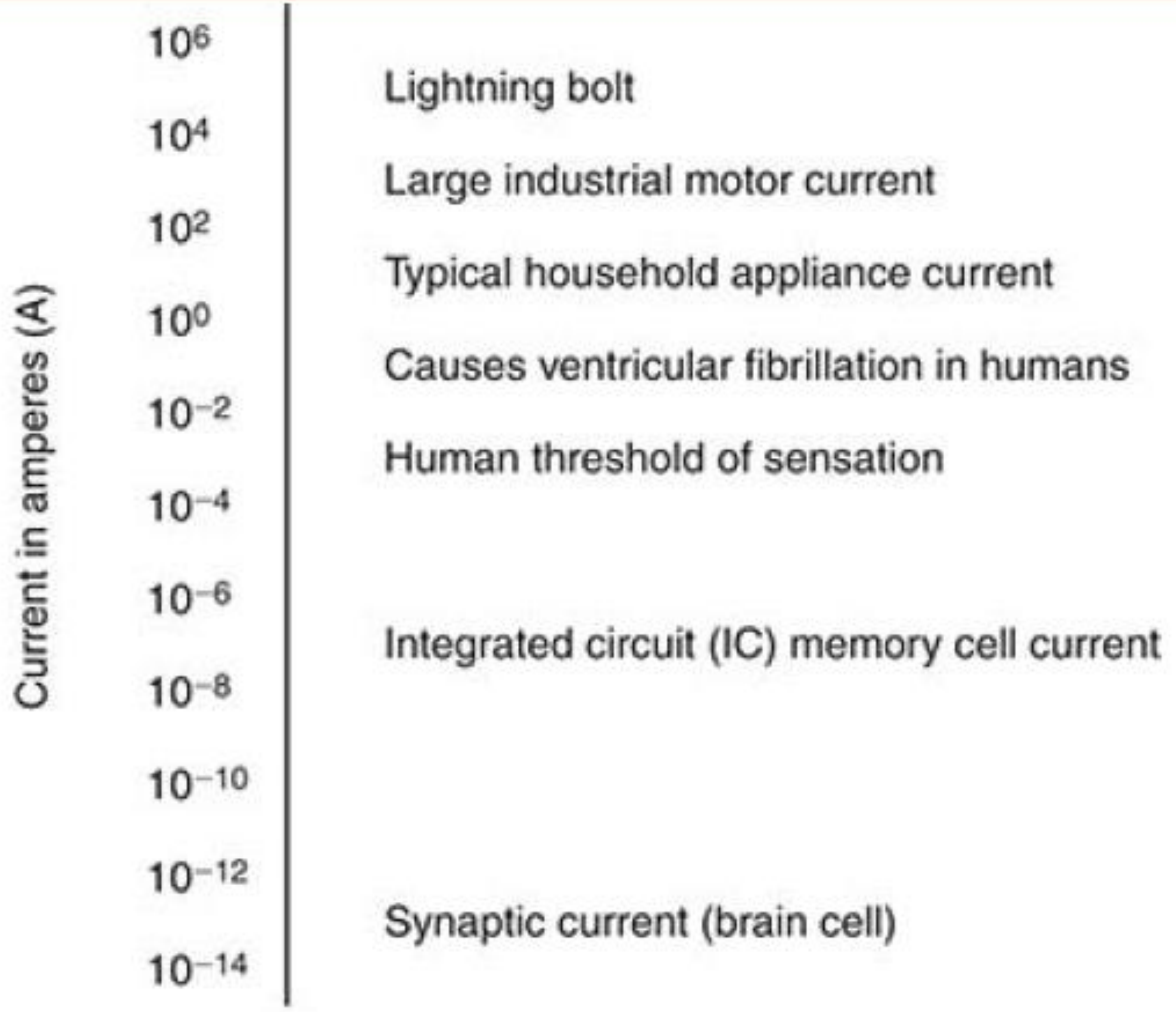
- In reality in metallic conductors current is due to the movement of **electrons**, however, we follow the universally accepted convention that current is in the direction of positive charge movement.



- Two ways of showing the same current:



Magnitude of Some Typical Currents



Voltage (Separation of Charge)

- **Voltage** (electromotive force, or potential) is the energy required to move a unit charge through a circuit element, and is measured in Volts (Alessandro Antonio Volta (1745-1827) an Italian Physicist).

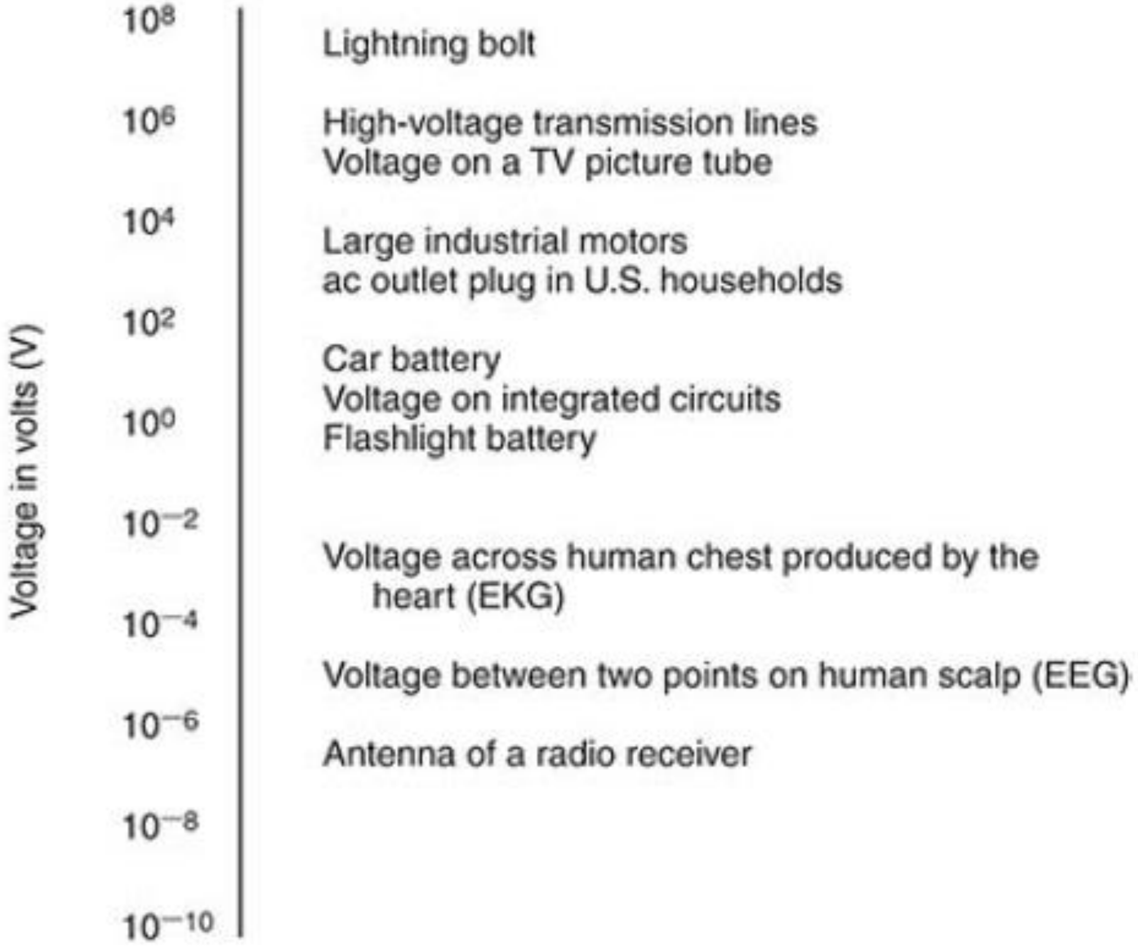
$$v = \frac{dw}{dq},$$

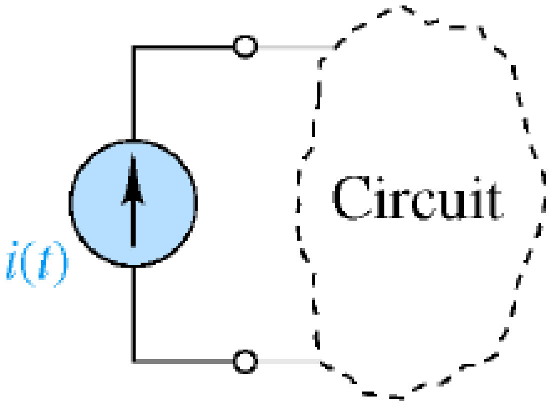
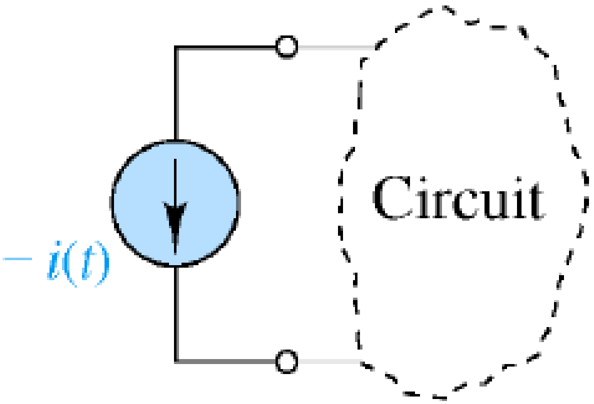
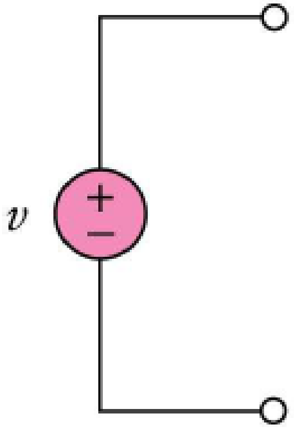
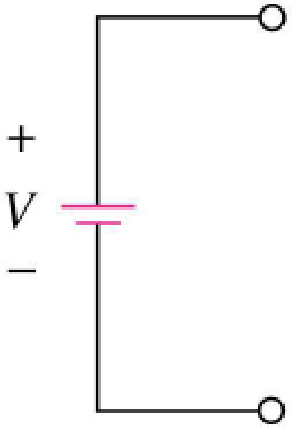
v = the voltage in volts,

w = the energy in joules,

q = the charge in coulombs.

Typical Voltage Magnitudes

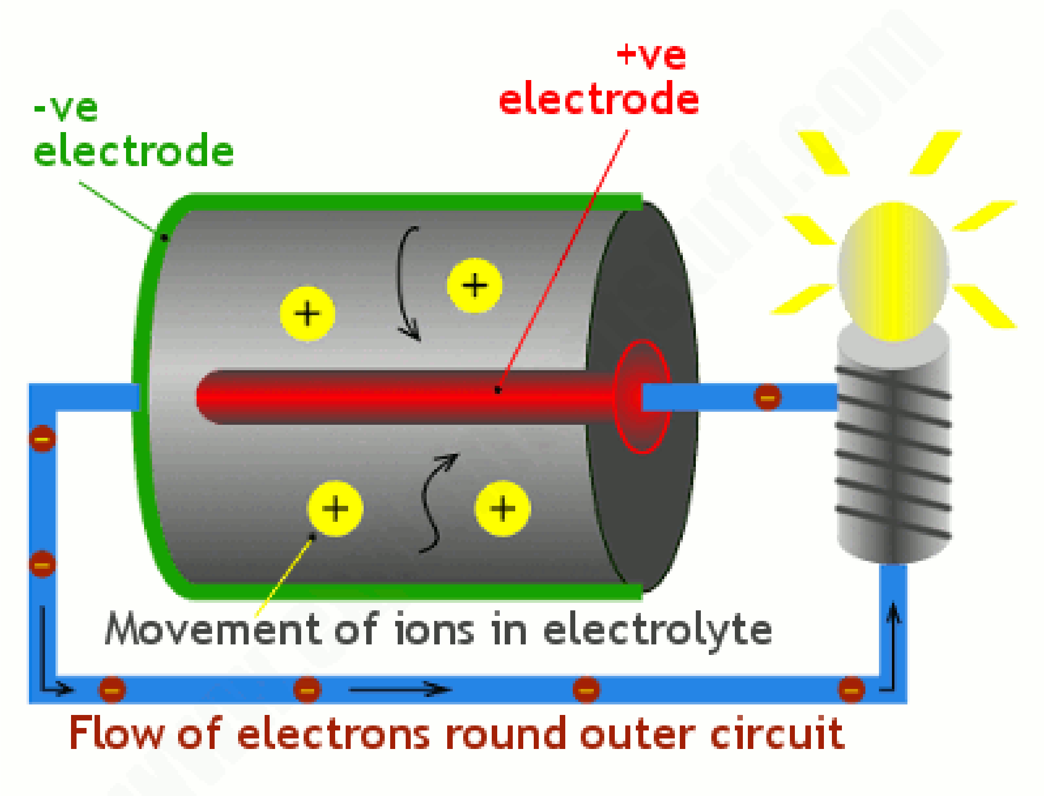



i(t)
Circuit

− i(t)
Circuit

v
+
−

+
V
−

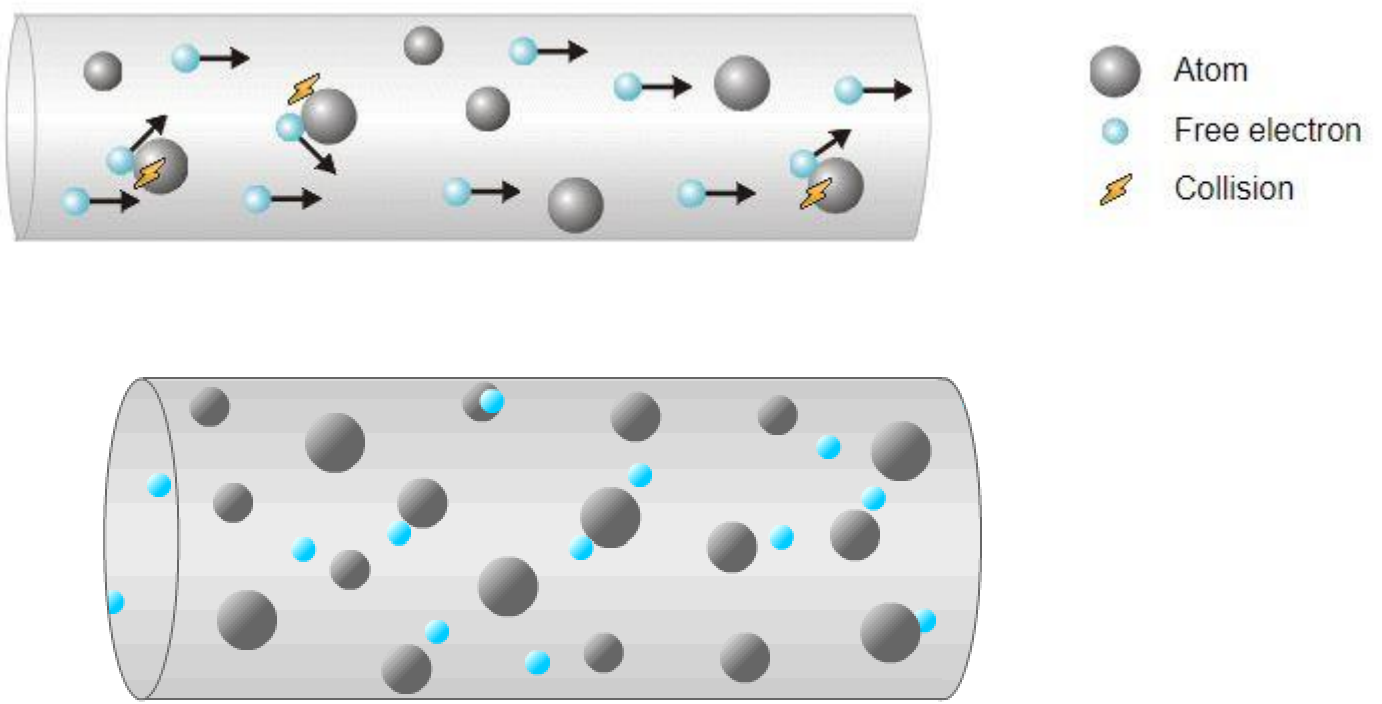
Five Basic Circuit Elements

1. Voltage Source (causes current flow)
2. Current Source (causes current flow)
3. Resistor (opposes current flow)
4. Capacitor (stores energy in electric field)
5. Inductor (opposes changes in current – i.e., its an AC resistor)

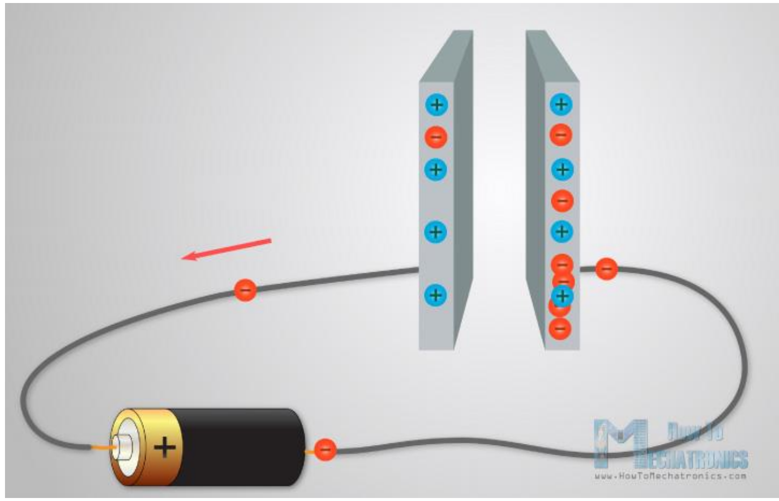
Voltage Source

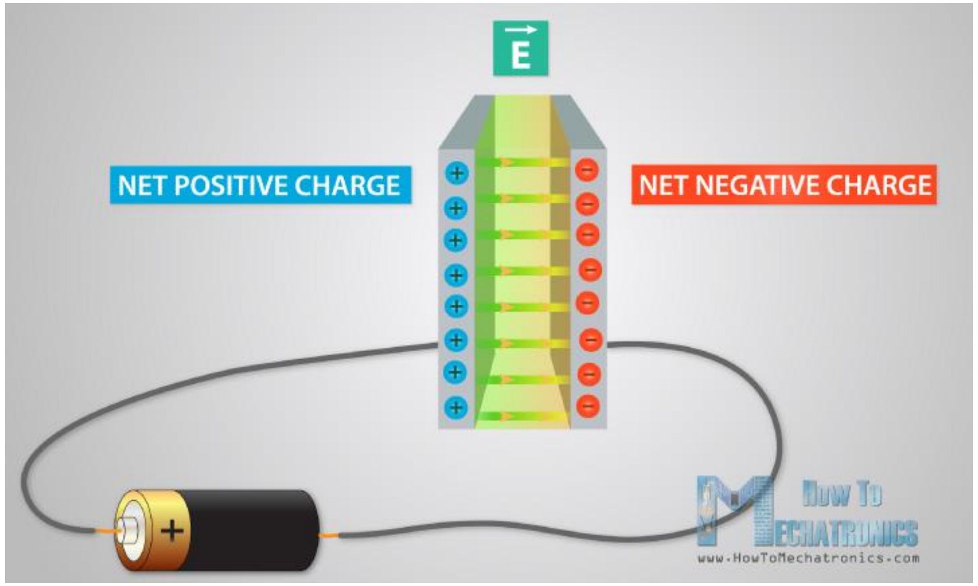


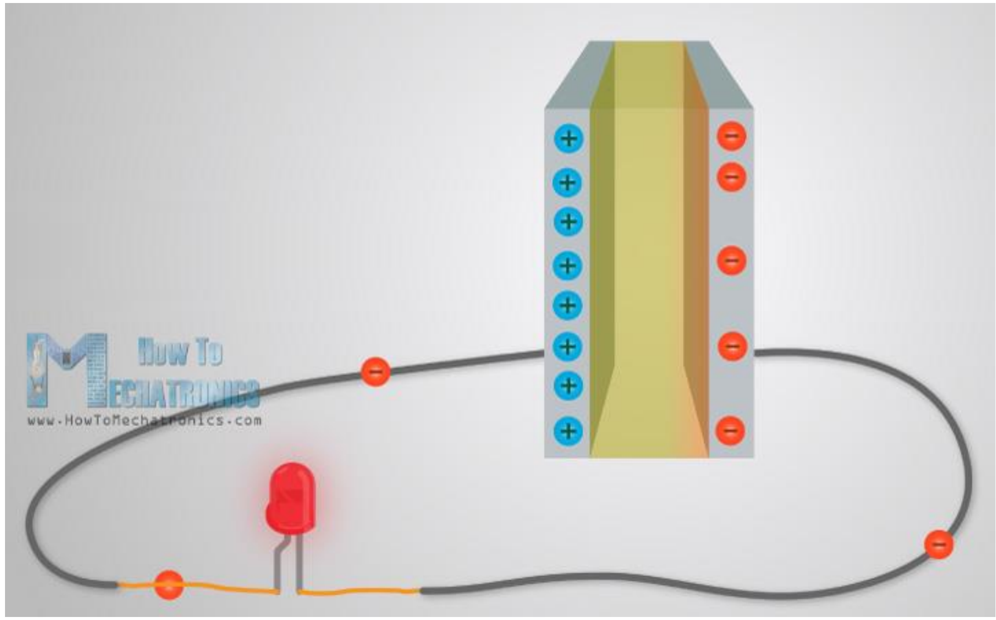
Resistance



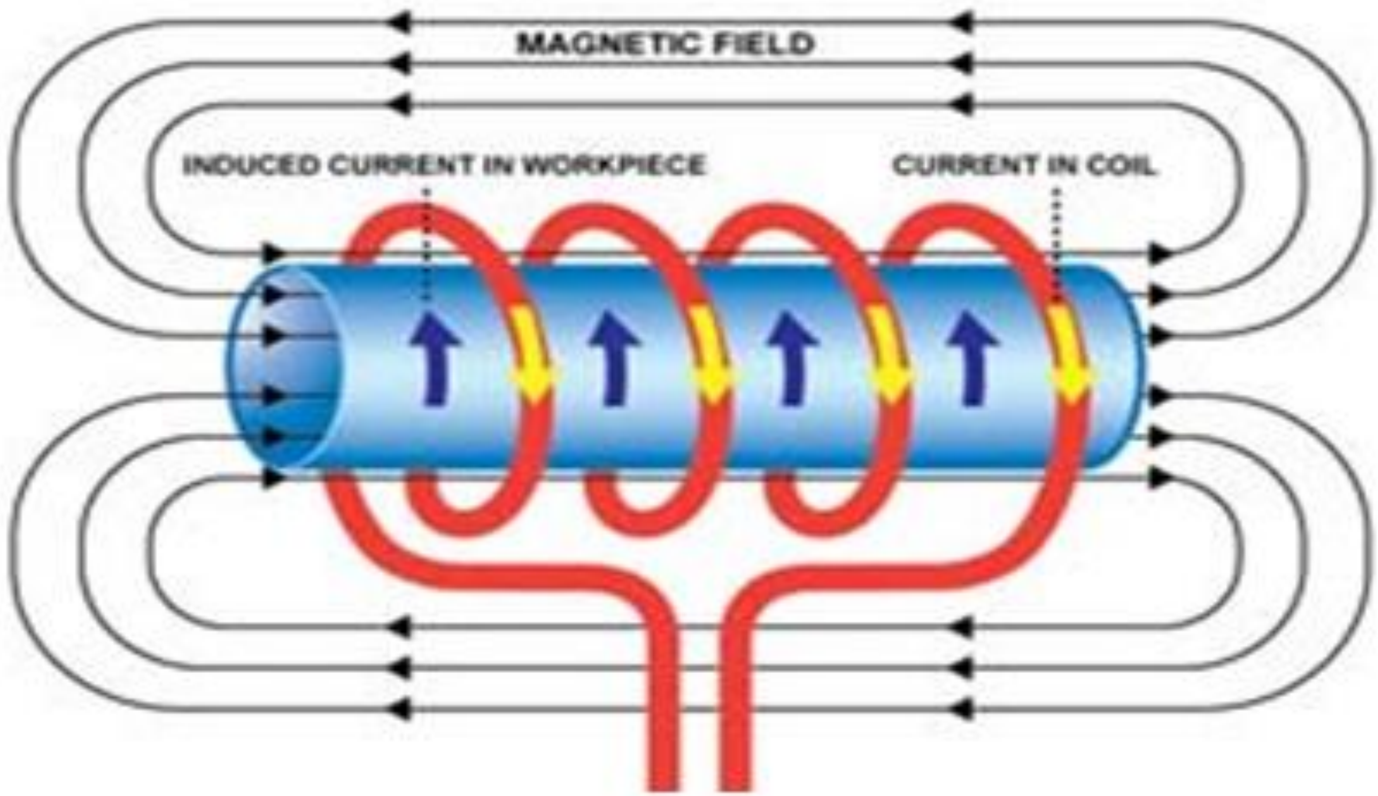
Capacitor




E
NET POSITIVE CHARGE
NET NEGATIVE CHARGE
How To
MECHATRONICS
www.HowToMechatronics.com


How To
MECHATRONICS
www.HowToMechatronics.com

Inductor

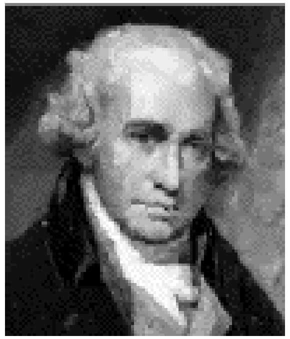

MAGNETIC FIELD
INDUCED CURRENT IN WORKPIECE
CURRENT IN COIL

Electrical Power and Energy

- Energy?
  - Ability to do work (e.g., kinetic energy, potential energy)
- Electrical Energy
  - Usually in form of potential (voltage) and kinetic (current) energies related to charges
- Power?
  - Rate of change of energy
- Electrical Power
  - Rate of change of electrical energy

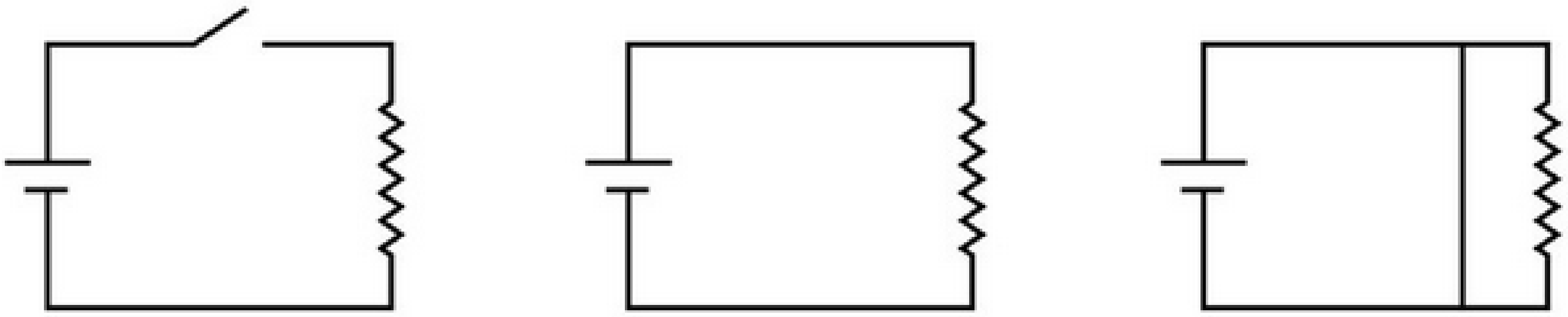
Power

- The rate of change of (expending or absorbing) energy per unit time, measured in Watts (James Watt (1736-1819) a Scottish inventor and mechanical engineer)

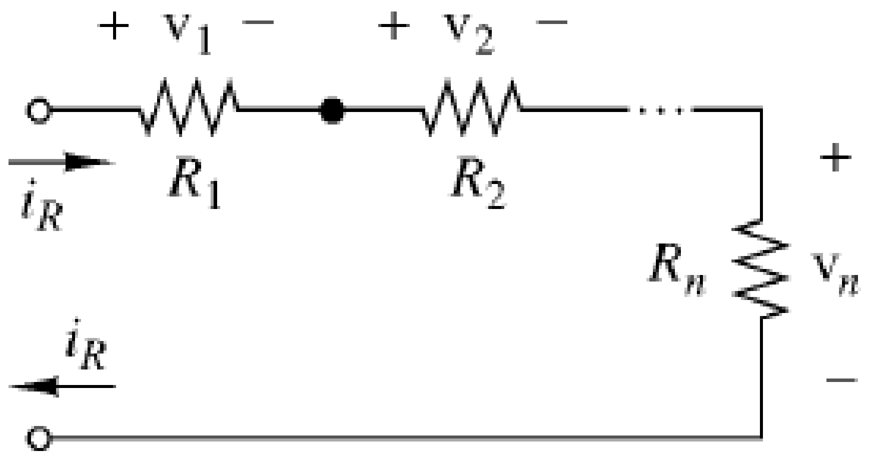


$$p = \frac{dW}{dt} = \frac{dW}{dq} \times \frac{dq}{dt} = vi$$

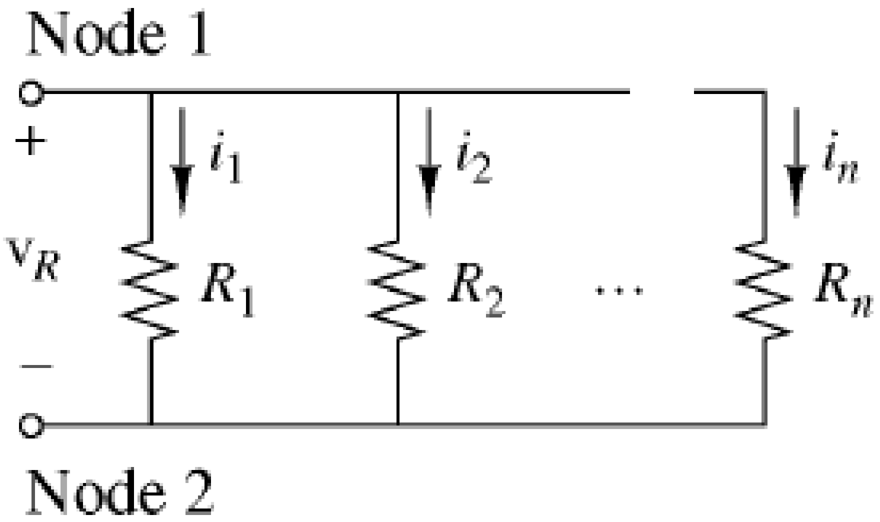
Circuits: open, closed, short



Circuits: Series vs Parallel



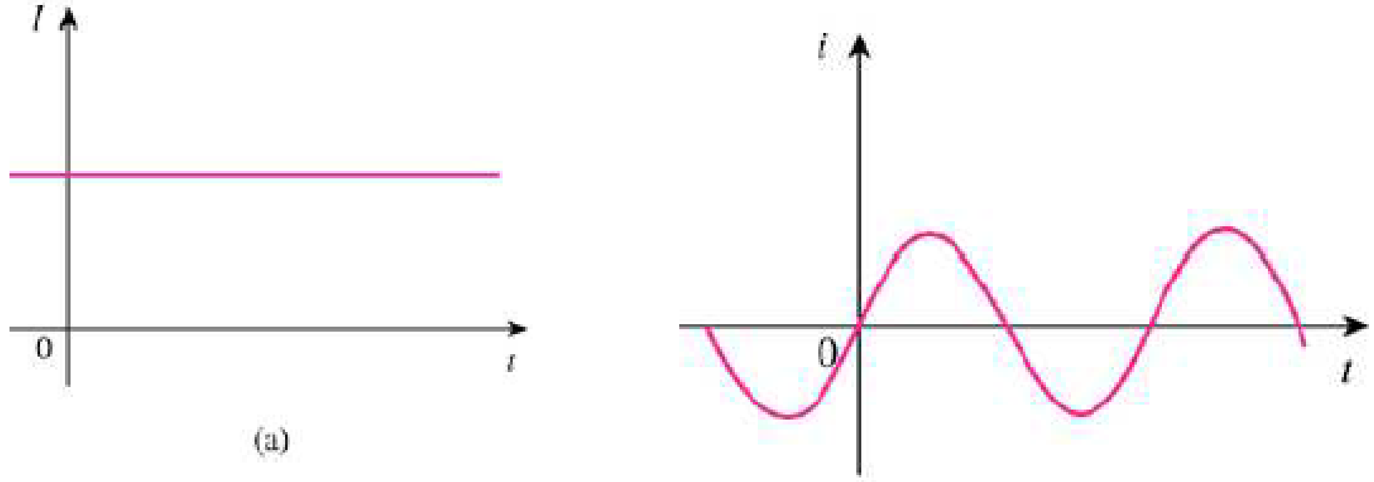
Elements that are in series carry the same current.



parallel elements have the same voltage

Current: DC vs AC

- Direct current (DC) is a current that remains constant with time.
- Alternating current (AC) is a current that varies sinusoidally with time.



(a) (b)

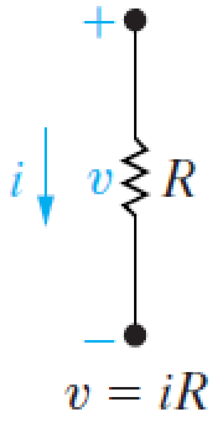
DC various elements

- How does each of these elements react when DC current passes through them?
  - Resistor
  - Inductor
  - Capacitor

DC through Resistor

Ohm's law ▶

$$v = iR,$$



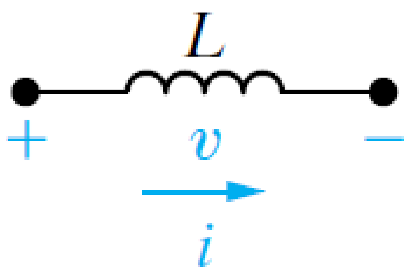
v = the voltage in volts,

i = the current in amperes,

R = the resistance in ohms.

DC through Inductor

$$v = L\frac{di}{dt},$$



v = the voltage in volts,

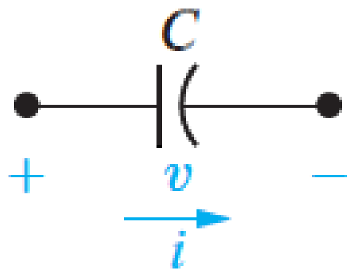
i = the current in amperes,

L = inductance in Henrys (H)

For DC, $v = 0$ since current does not change!! (i.e. for DC, inductor behaves as a short circuit)

DC through Capacitor

$$i = C\frac{dv}{dt},$$

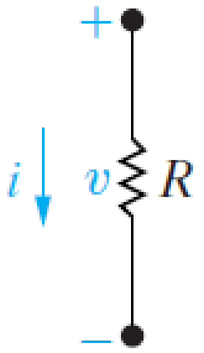


C = capacitance in Farads (F)

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i\, d\tau + v(t_0).$$

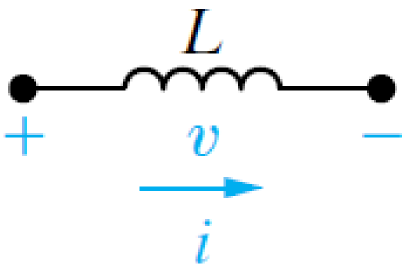
For DC, capacitor charges/discharges until no more current flows through it!! (i.e. for DC, capacitor eventually behaves as an open circuit)

How to find power consumed by the elements? Use: $p = vi.$

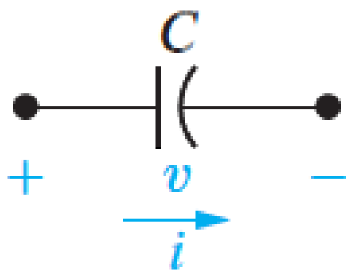


$$p = i^2 R.$$

$$p = \frac{v^2}{R}.$$



$$p = Li\frac{di}{dt}.$$

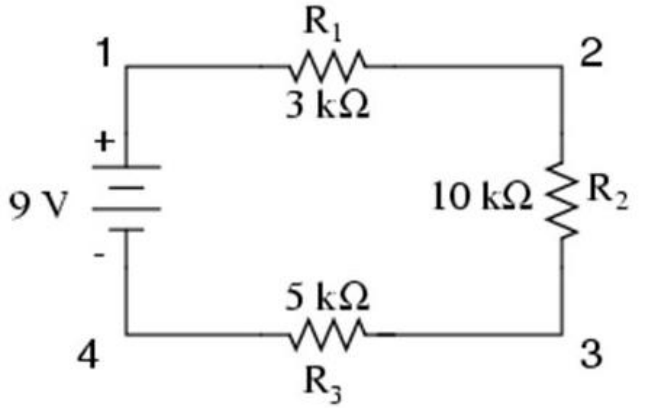


$$p = vi = Cv\frac{dv}{dt},$$

Quick Revision I – Equivalent Resistance

Series circuits

$$R_{eq} = R_1 + R_2 + ... + R_n$$

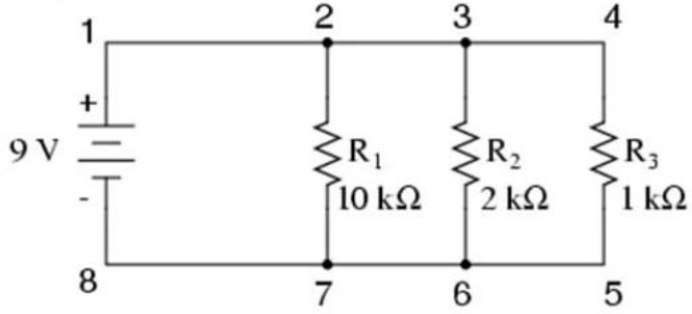


3kΩ + 10kΩ + 5kΩ = 18kΩ

Req = 18kΩ

in a Parallel Circuit

$$1/R_{eq} = 1/R_1 + 1/R_2 + ... + 1/R_n$$

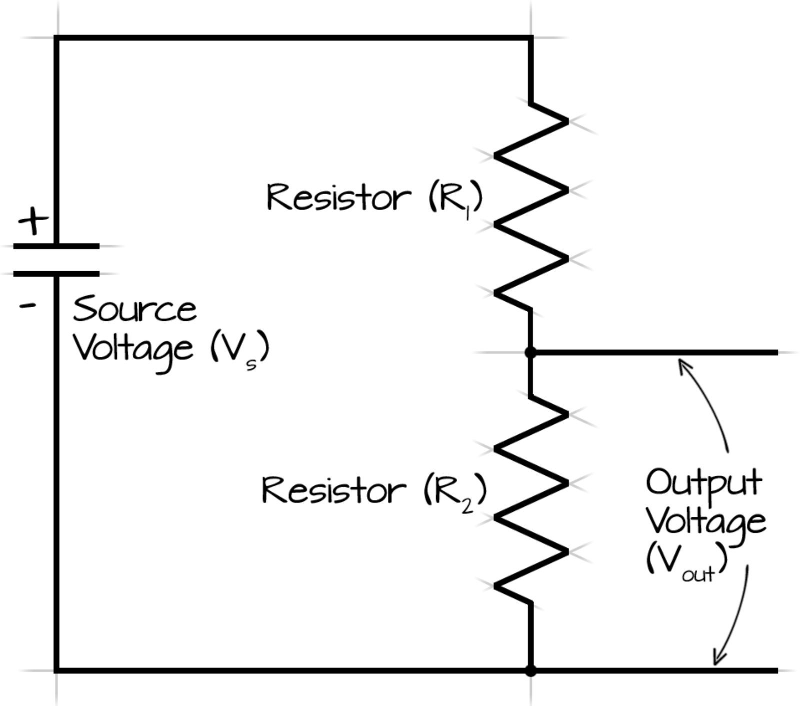


$1/R_{eq}$ = 1/10Ω +1/2Ω + 1/1 Ω

1/Req = 0.1Ω + 0.5Ω + 1Ω = 1.6Ω

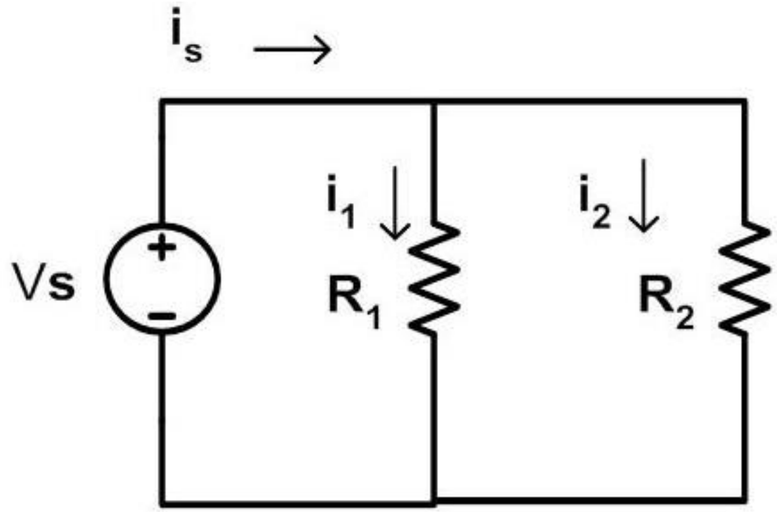
R_{eq} = 1/ 1.6 Ω

Quick Revision II – Divider Rules



$$V_{out} = \frac{V_s \times R_2}{(R_1 + R_2)}$$

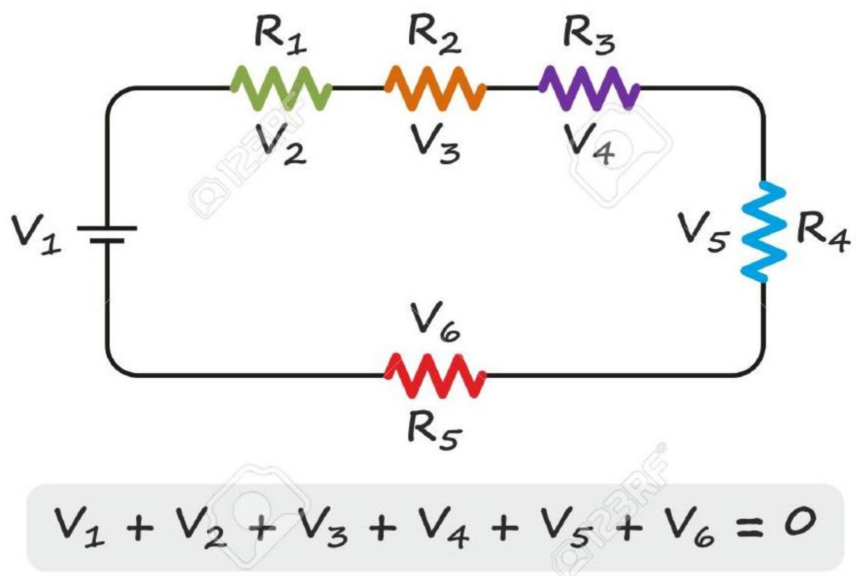
Quick Revision II – Divider Rules

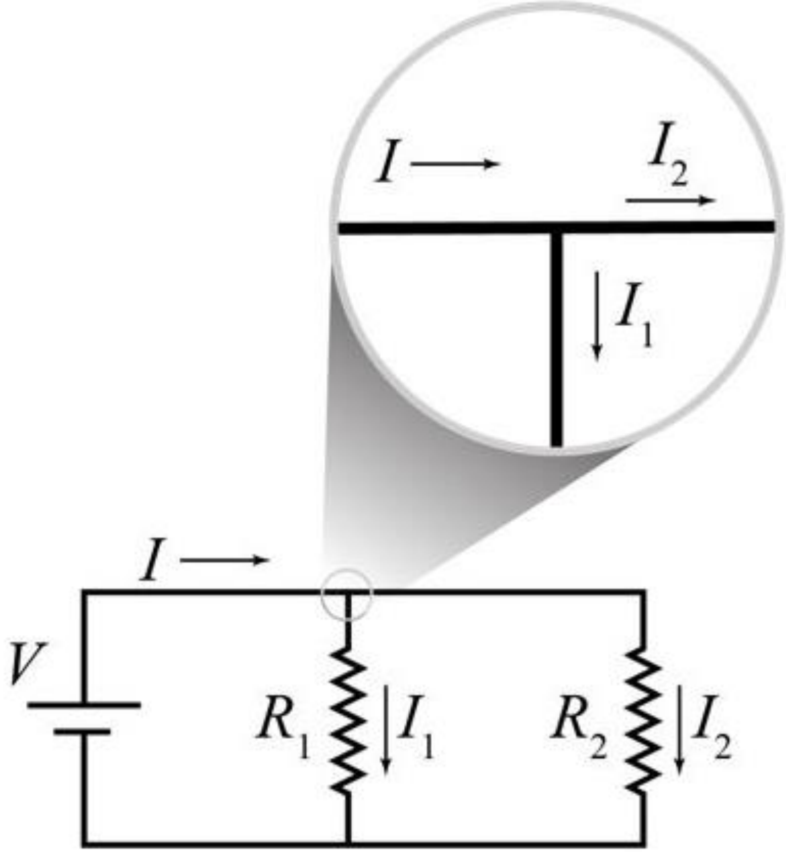


$$i_1 = \frac{R_2}{R_1 + R_2} i_s$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_s$$

Quick Revision III – Loop & Branch Rules





Examples

Questions?? Thoughts??

EE 202
Electric Circuit Analysis

with

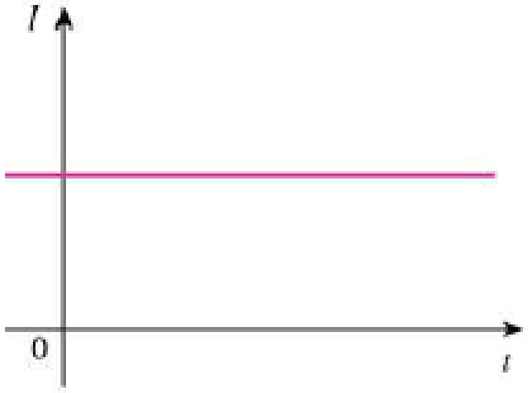
Dr. Naveed R. Butt

@

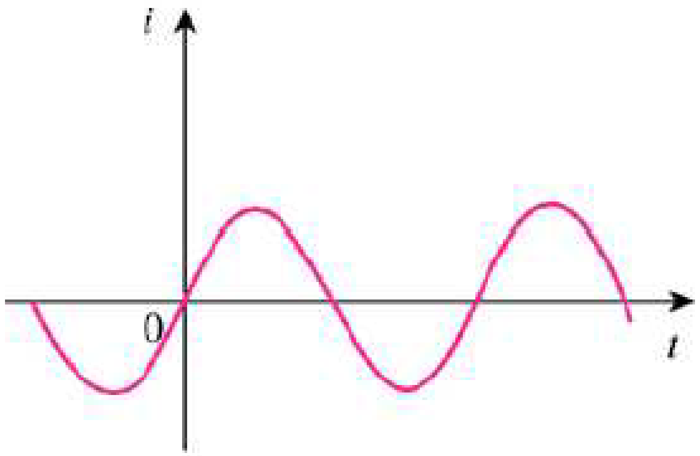
Jouf University

Current: DC vs AC

- Direct current (DC) is a current that remains constant with time.
- Alternating current (AC) is a current that varies sinusoidally with time.



(a)



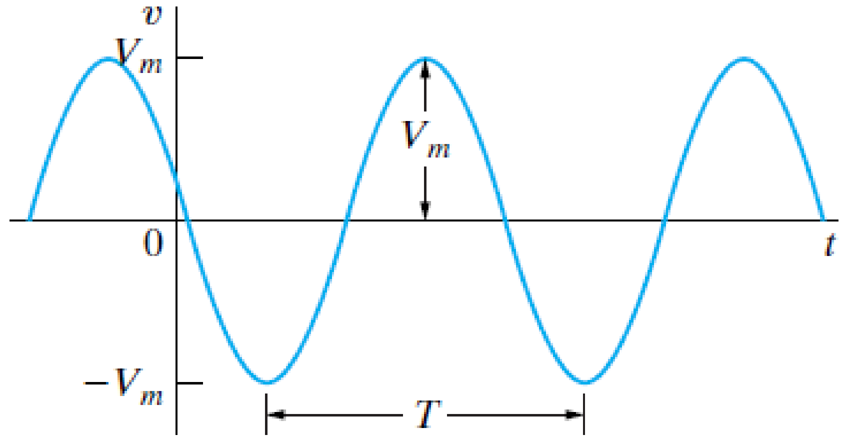
(b)

AC: important parameters

$$v = V_m \cos(\omega t + \phi).$$

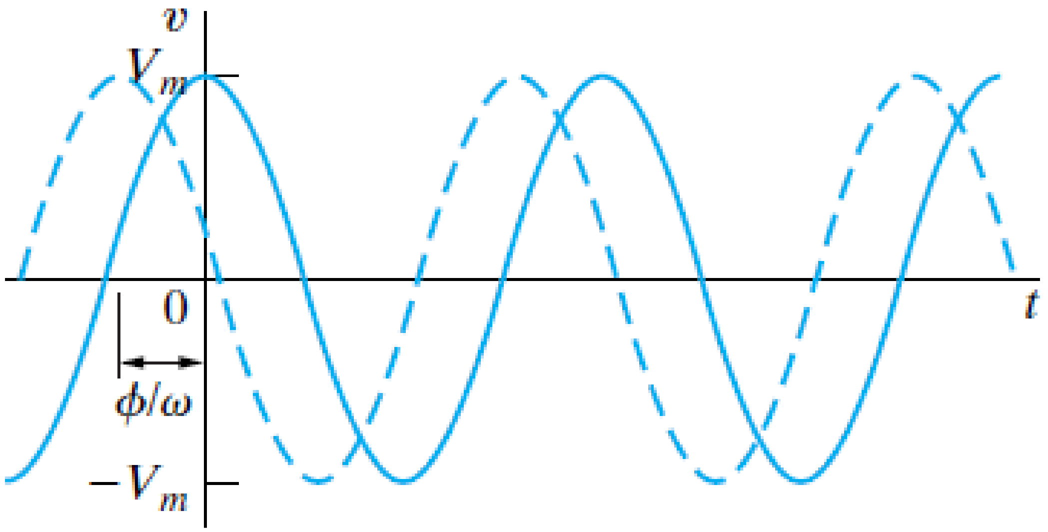
$$\omega = 2\pi f$$

$$f = \frac{1}{T}.$$



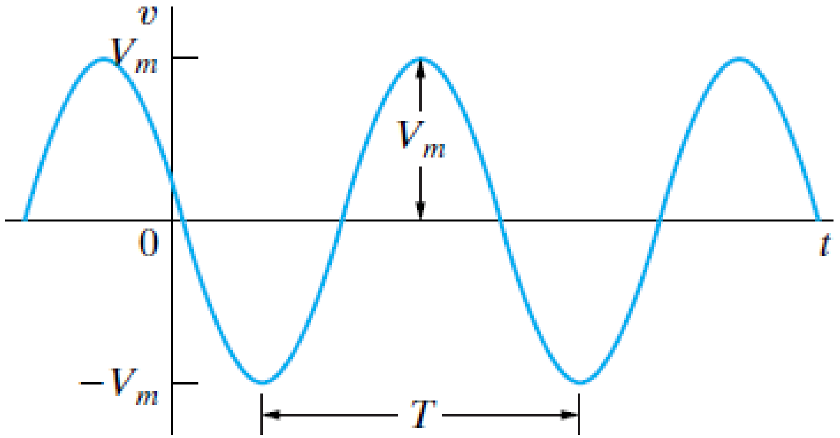
AC: important parameters

- Peak Value
- Frequency
- Phase
- rms-value



AC: important parameters

- Peak Value
- Frequency
- Phase
- rms-value



$$f = \frac{1}{T}.$$

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}.$$

AC: important parameters

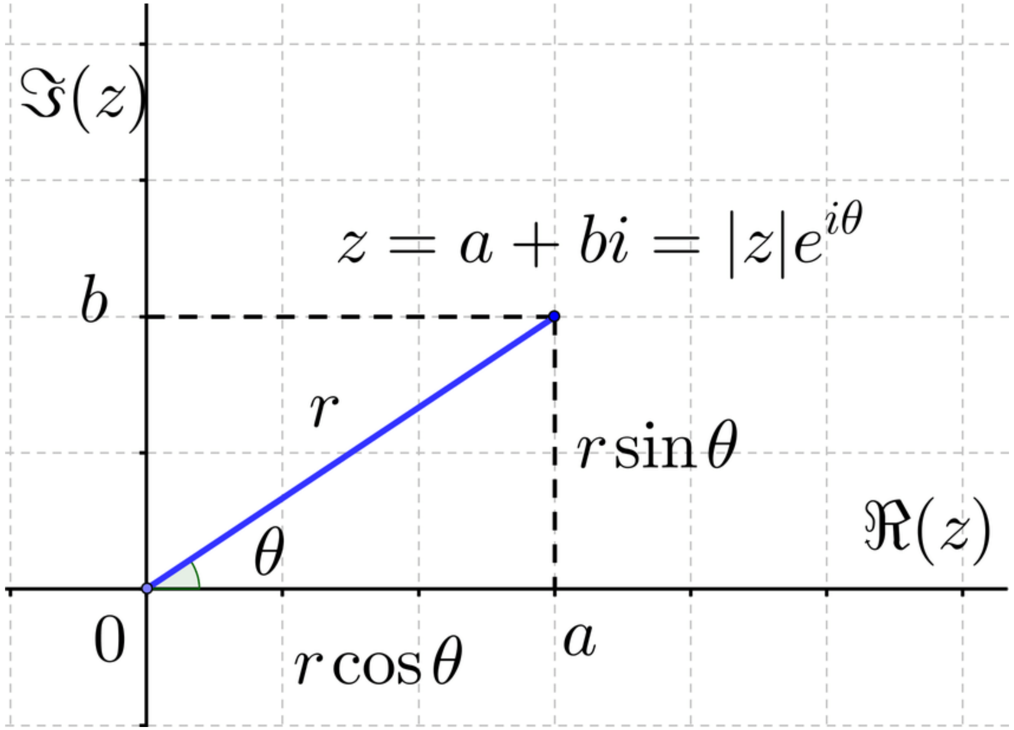
Similarly, for sinusoidal current we have …

$$i(t) = I_m \cos(\omega t + \phi)$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Practice Examples

Complex Number in Cartesian and Polar Form



Cartesian form $z = a + jb$

Polar form $z = |z|e^{j\theta}$

Cartesian to Polar

$$|z| = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

Polar to Cartesian

$$a = |z|\cos\theta$$
$$b = |z|\sin\theta$$

Phasor Notation

Note: phasor notation does not retain frequency information!!

$$\mathbf{V} = V_m e^{j\phi} = \mathcal{P}\{V_m \cos(\omega t + \phi)\},$$

$$V_m e^{j\phi} \equiv V_m \underline{/\phi^\circ}$$

$$\mathbf{V} = V_m \cos\phi + jV_m \sin\phi.$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta.$$

$$\cos\theta = \Re\{e^{j\theta}\},$$

$$\sin\theta = \Im\{e^{j\theta}\},$$

$$\sin(\theta + \tfrac{\pi}{2}) = +\cos\theta$$

Extracting Phasor from Cartesian Form

$$\mathbf{V} = a + jb$$

Convert to polar form!!

$$V_m^2 = a^2 + b^2$$

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\mathbf{V} = V_m e^{j\phi}$$

using

$$\mathbf{V} = V_m \cos\phi + jV_m \sin\phi.$$

Multiplying Phasors

$$\mathbf{V_1} = V_{m_1} e^{j\phi_1}$$
$$\mathbf{V_2} = V_{m_2} e^{j\phi_2}$$

$$\mathbf{V_1 V_2} = V_{m_1} V_{m_2} e^{j(\phi_1 + \phi_2)}$$

$$= V_{m_1} V_{m_2} \angle(\phi_1 + \phi_2)$$

Adding Phasors

$$\mathbf{V_1} = V_{m_1} e^{j\phi_1}$$
$$\mathbf{V_2} = V_{m_2} e^{j\phi_2}$$

$$\mathbf{V_1}+\mathbf{V_2}= V_{m_1}\cos\phi_1 + V_{m_2}\cos\phi_2 + j\left(V_{m_1}\sin\phi_1 + V_{m_2}\sin\phi_2\right)$$

Finally, extract phasor from this complex number (as described previously)

Practice Examples

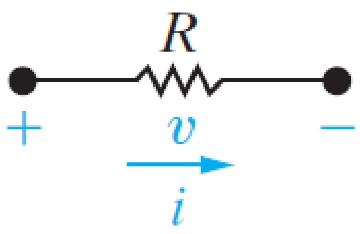
AC through various elements

- How does each of these elements react when AC current passes through them?
  - Resistor
  - Inductor
  - Capacitor

$$\mathbf{V} = \mathbf{ZI},$$

Unified notation!
Z = Impedance

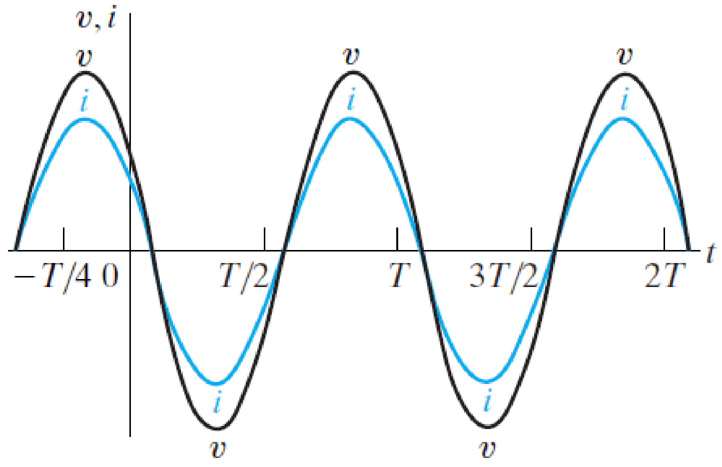
AC through Resistor



$$\mathbf{V} = RI_m e^{j\theta_i} = RI_m \angle \theta_i.$$

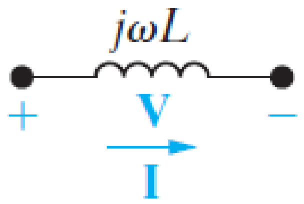
$$\mathbf{V} = R\mathbf{I},$$

$$\theta_v = \theta_i$$



When AC passes through resistor, the resulting voltage has same phase as the current.

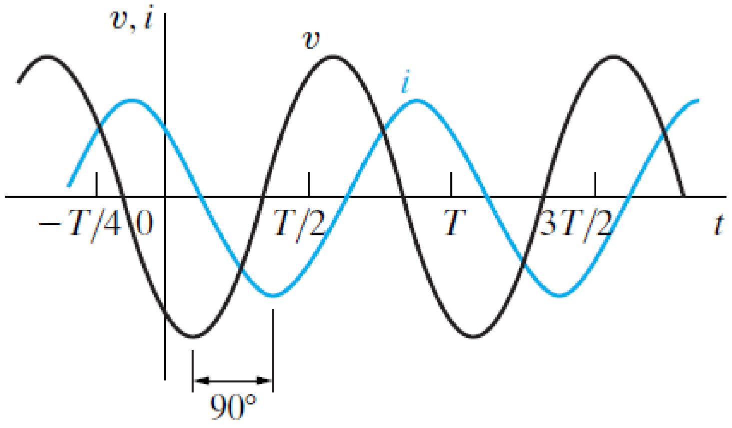
AC through Inductor



$$\mathbf{V} = \omega L I_m \angle(\theta_i + 90)^\circ,$$

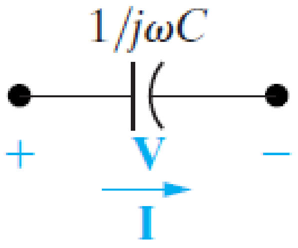
$$\mathbf{V} = j\omega L\mathbf{I}.$$

$$\theta_v = \theta_i + 90^o$$

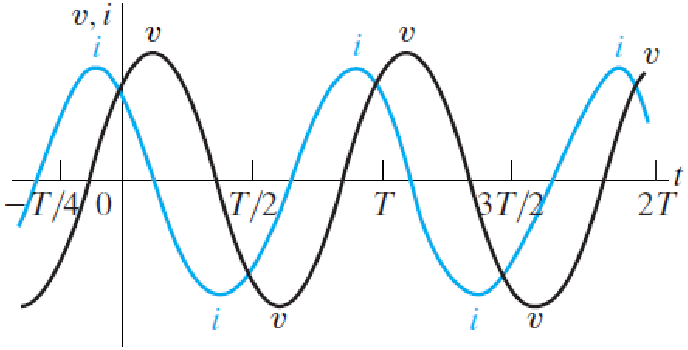


When AC passes through Inductor, the resulting voltage leads the current by 90 degrees.

AC through Capacitor



$$\mathbf{V} = \frac{I_m}{\omega C} \angle(\theta_i - 90)^\circ.$$



$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}.$$

When AC passes through Capacitor, the resulting voltage lags behind the current by 90 degrees.

$$\theta_v = \theta_i - 90^{\text{o}}$$

Unified Notation

$$\mathbf{V} = \mathbf{ZI},$$

Resistor: $\mathbf{V} = R\mathbf{I},$ $\quad Z = R$

Inductor: $\mathbf{V} = j\omega L\mathbf{I}.$ $\quad Z = j\omega L$

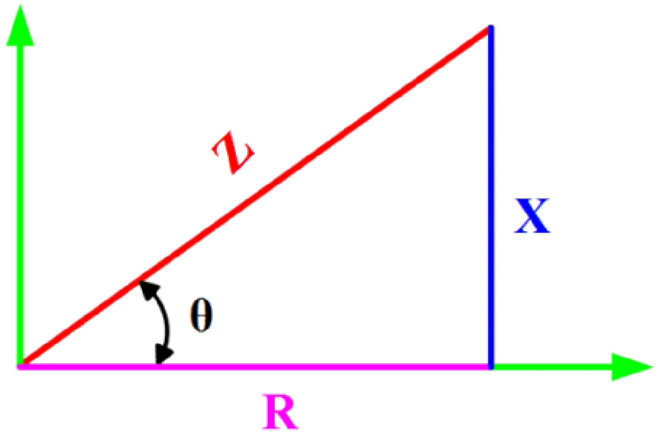
Capacitor: $\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}.$ $\quad Z = \frac{1}{j\omega C} = j(\frac{-1}{\omega C})$

Resistance, Impedance, Reactance

When dealing with **DC** we talk about **Resistance**, but when dealing with AC we need **Impedance**!!

Complex Form	**Real Part**	**Imaginary Part**
Impedance (Z)	Resistance (R)	Reactance (X)

$$Z = R + jX$$



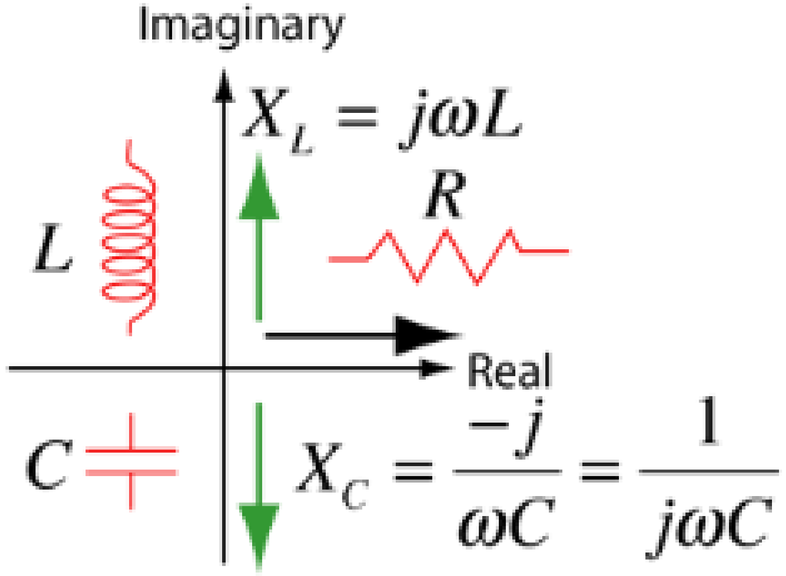
Resistors: have R but no X
Capacitors and Inductors: have X but no R

Units

Resistance, Impedance, and Reactance are all measured in Ohms (Ω)

Resistance, Impedance, Reactance

Element	Impedance	Resistance (real part of Impedance)	Reactance (imaginary part of Impedance)
Resistor	$\boldsymbol{R}$	$\boldsymbol{R}$	–
Inductor	$\boldsymbol{j\omega L}$	–	$\boldsymbol{\omega L}$
Capacitor	$\boldsymbol{j\frac{-1}{\omega C}}$	–	$\boldsymbol{\frac{-1}{\omega C}}$



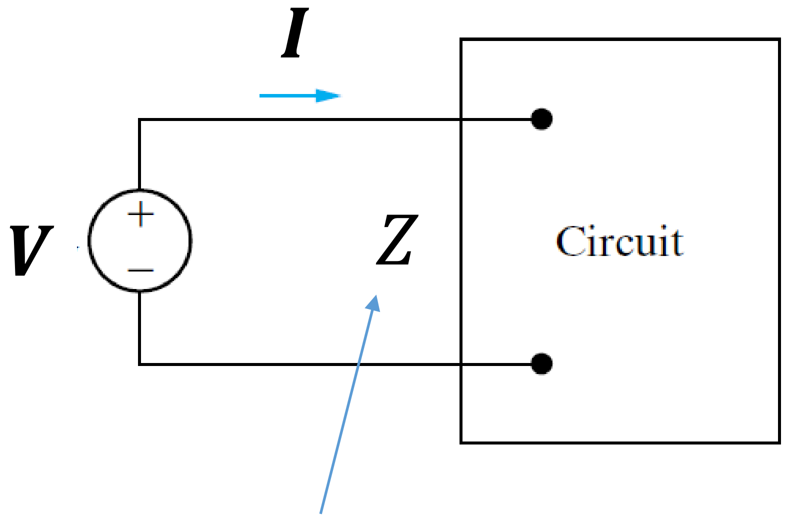
Combined Impedance (General Formula)

- When we combine elements in different ways in circuits, we get different combined impedances (leading also to different overall phase shifts). General formulation is as follows.

$$\boldsymbol{V} = Z\boldsymbol{I}$$ where $$Z = |Z|e^{j\phi_Z}$$

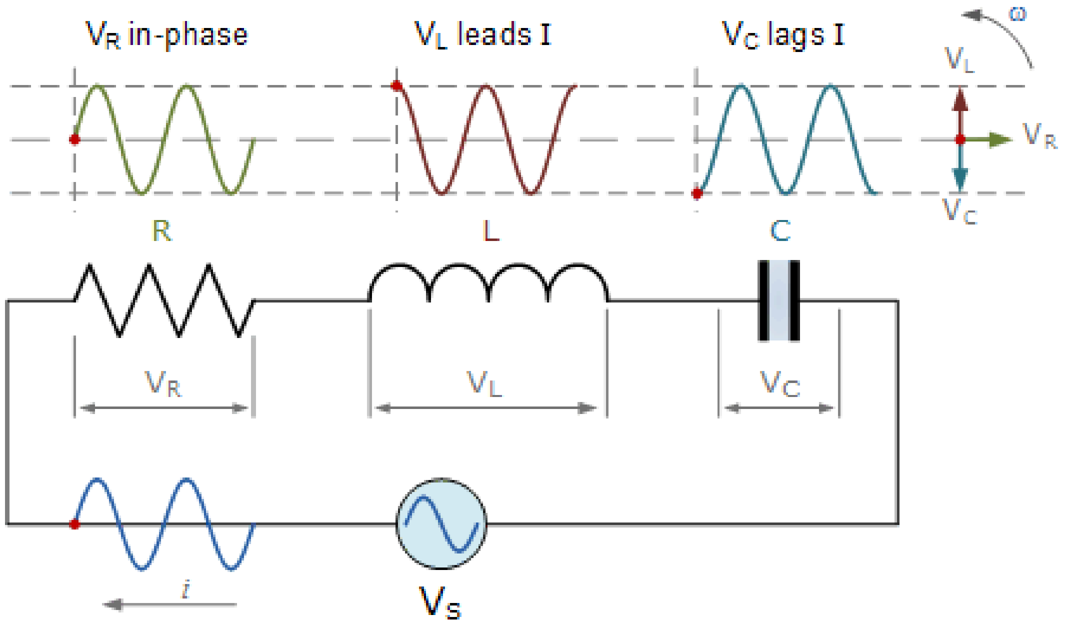
then

$$\boldsymbol{V} = |Z|I_m e^{j(\phi_i+\phi_Z)}$$



Total (combined) impedance of the circuit

Combined Impedance (example: RLC)



$$\boldsymbol{V_s} = Z\boldsymbol{I}$$

$$\boldsymbol{V_s} = (Z_R + Z_L + Z_C)\boldsymbol{I}$$

$$\boldsymbol{V_s} = (R + j\omega L - j\frac{1}{\omega C})\boldsymbol{I}$$

$$\boldsymbol{V_s} = (R + j\omega L - j\frac{1}{\omega C})\boldsymbol{I}$$

$$\boldsymbol{V_s} = (R + jX_L - jX_C)\boldsymbol{I}$$

Combined Impedance (example: RLC)

$$\boldsymbol{V_s} = (R + jX_L - jX_C)\boldsymbol{I}$$

$$\boldsymbol{V_s} = |Z|e^{j\phi_Z}\boldsymbol{I}$$

$$\boldsymbol{V_s} = |Z|e^{j\phi_Z}I_m e^{j\phi_i} = |Z|I_m e^{j(\phi_i+\phi_Z)}$$

Converting part in parenthesis to polar form, with

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi_Z = \tan^{-1}\frac{X_L - X_C}{R}$$

Practice Examples

Questions?? Thoughts??

EE 202
Electric Circuit Analysis

with

Dr. Naveed R. Butt

@

Jouf University

Recall: DC → AC

$$Z = R + jX$$

$$Z = |Z|e^{j\theta}$$

DC

Real Numbers

$$v = iR,$$

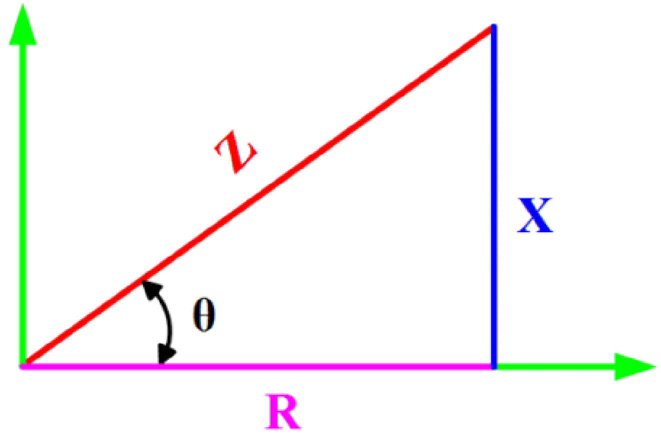
Resistance

AC

Complex Numbers

$$\mathbf{V} = Z\mathbf{I},$$

Impedance



$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta = \tan^{-1}\frac{X}{R}$$

Introducing: Complex Power (S)

$$S = P + jQ.$$

$$S = |S|e^{j\theta}$$

DC

Real Numbers

$$p = vi.$$

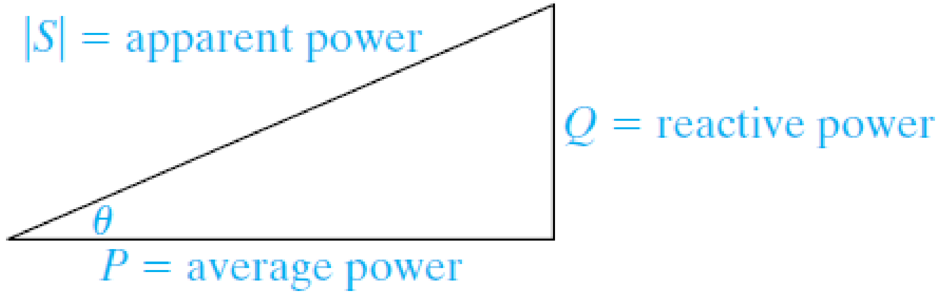
Real Power

AC

Complex Numbers

$$S = \frac{1}{2}\mathbf{V}\mathbf{I}^*.$$

Complex Power



$$|S| = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1}\frac{Q}{P}$$

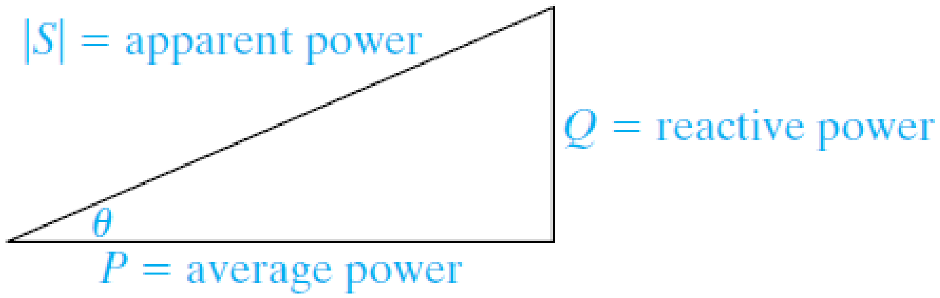
Complex Conjugate

To get the complex conjugate, just change the sign of *j*

$$\mathbf{I} = I_m e^{j\theta} \quad \longleftrightarrow \quad \mathbf{I}^* = I_m e^{-j\theta}$$

$\mathbf{I}$ *and* $\mathbf{I}^*$ are complex conjugates of each other

Five Powers



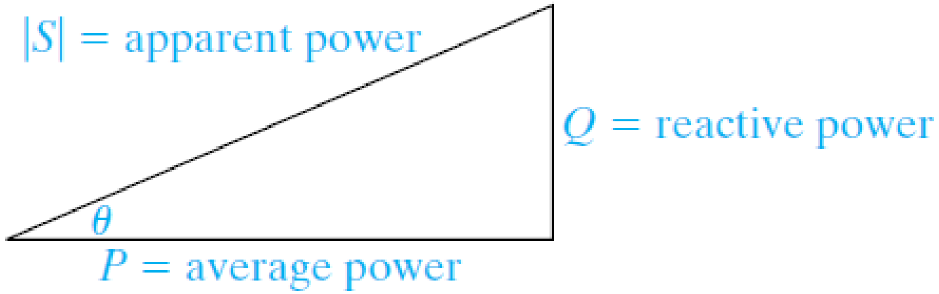
Power Name	Symbol	Units
Complex Power	S	Volt-Amps (va)
Apparent Power	$\|S\|$	Volt-Amps (va)
Average Power	P	Watts
Reactive Power	Q	Volt-Amps-Reactive (var)
Instantaneous Power	$p(t)$ or p	Watts

$$P = |S| \cos \theta,$$

$$Q = |S| \sin \theta.$$

Power Factor Angle and Power Factor

It can be shown that the angle θ in the power triangle is in fact equal to $\theta_v - \theta_i$



Name	Formula
Power Factor Angle	$\theta = \theta_v - \theta_i = \tan^{-1}\dfrac{Q}{P}$
Power Factor	$\cos(\theta_v - \theta_i)$

Note that since $Z = \frac{\mathbf{V}}{\mathbf{I}}$, we can deduce that angle of impedance Z is the same as the power factor angle (i.e., Z and S have the same angle, $\theta_v - \theta_i$)

P = Average Power (can be calculated from several formulas)

$$S = P + jQ.$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i),$$

$$\tan(\theta_v - \theta_i) = \frac{Q}{P}$$

$$P = V_{\text{eff}} I_{\text{eff}} \cos(\theta_v - \theta_i);$$

$V_{eff} = V_{rms}$ and $I_{eff} = I_{rms}$

Q = Reactive Power (can be calculated from several formulas)

$$S = P + jQ.$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i).$$

$$\tan(\theta_v - \theta_i) = \frac{Q}{P}$$

$$Q = V_{\text{eff}} I_{\text{eff}} \sin(\theta_v - \theta_i).$$

$V_{eff} = V_{rms}$ and $I_{eff} = I_{rms}$

S = Complex Power (can be calculated from several formulas)

$$S = \frac{1}{2}\mathbf{V}\mathbf{I}^*. \quad = \frac{1}{2}V_m I_m \underline{/(\theta_v - \theta_i)}.$$

$$S = P + jQ.$$

$$S = \mathbf{V}_{\text{eff}}\mathbf{I}^*_{\text{eff}}. \quad = V_{\text{eff}} I_{\text{eff}} \underline{/(\theta_v - \theta_i)}.$$

$$S = I^2_{\text{eff}} Z = \frac{V^2_{\text{eff}}}{Z^*}.$$

$V_{eff} = V_{rms}$ and $I_{eff} = I_{rms}$

$|S|$ = Apparent Power (can be calculated from several formulas)

$$|S| = \sqrt{P^2 + Q^2}$$

$$S = |S|e^{j(\theta_v - \theta_i)}$$

$$P = |S|\cos(\theta_v - \theta_i)$$

$$Q = |S|\sin(\theta_v - \theta_i)$$

p = Instantaneous Power (can be calculated from following formula)

$$p = P + P\cos 2\omega t - Q\sin 2\omega t,$$

Practice Examples

Questions?? Thoughts??

EE 202
Electric Circuit Analysis

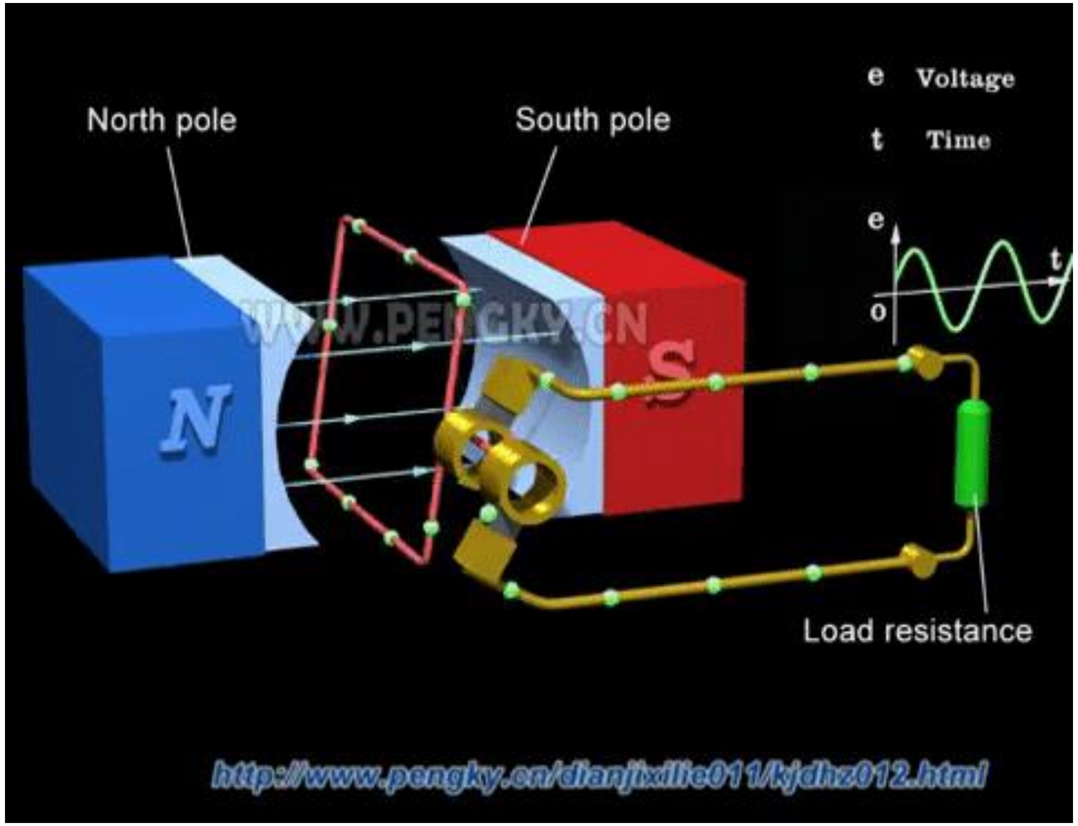
with

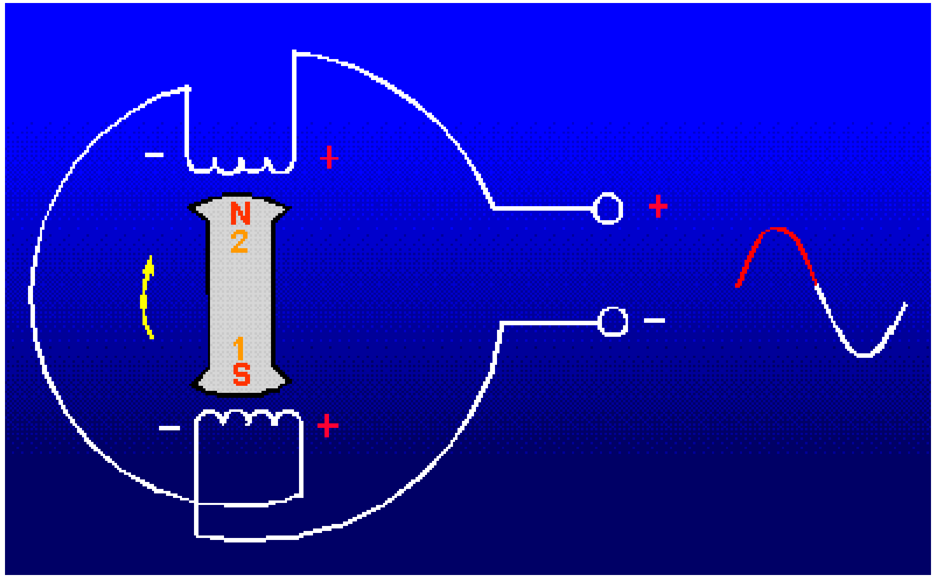
Dr. Naveed R. Butt

@

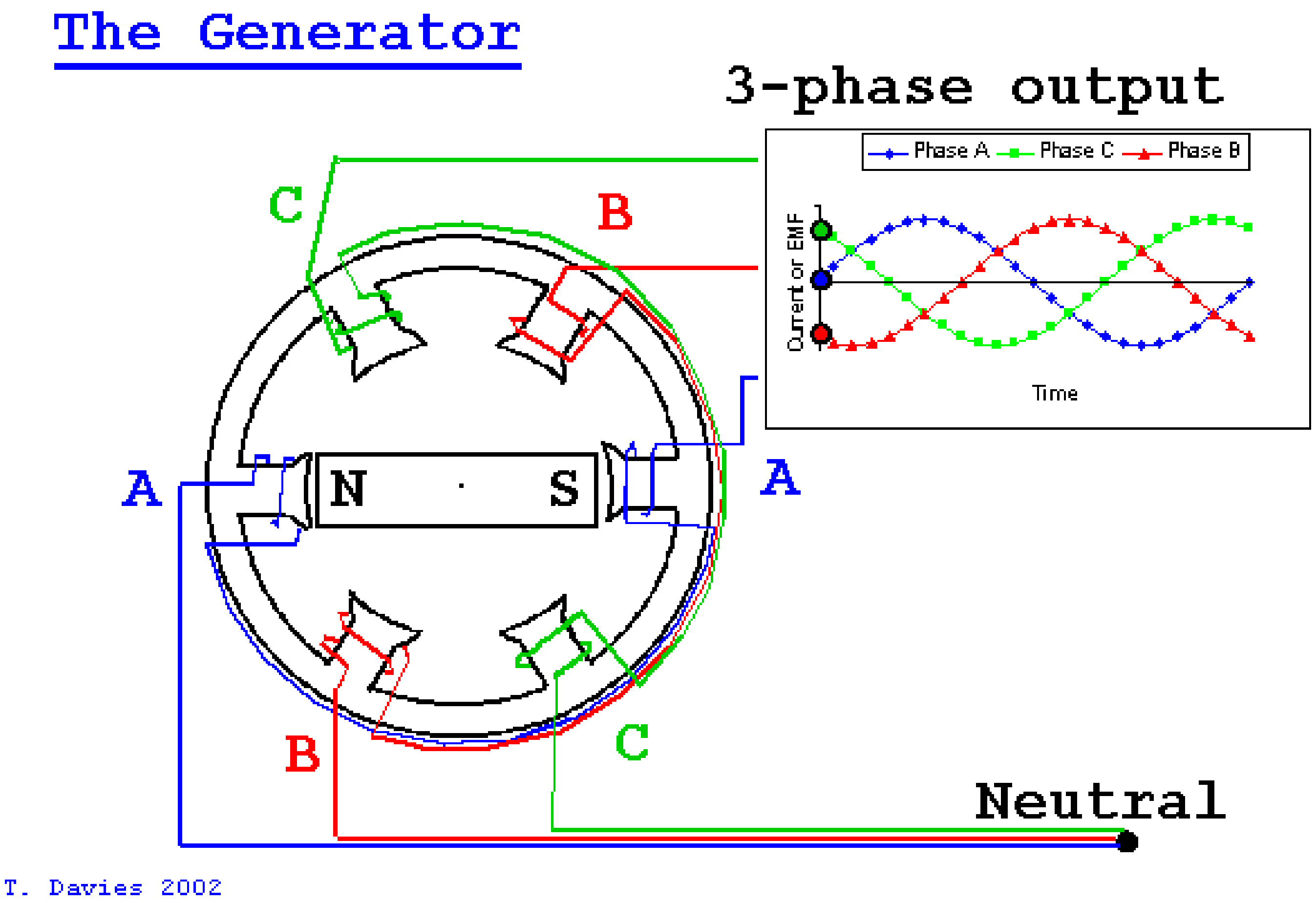
Jouf University

Single-Phase Electricity Generation



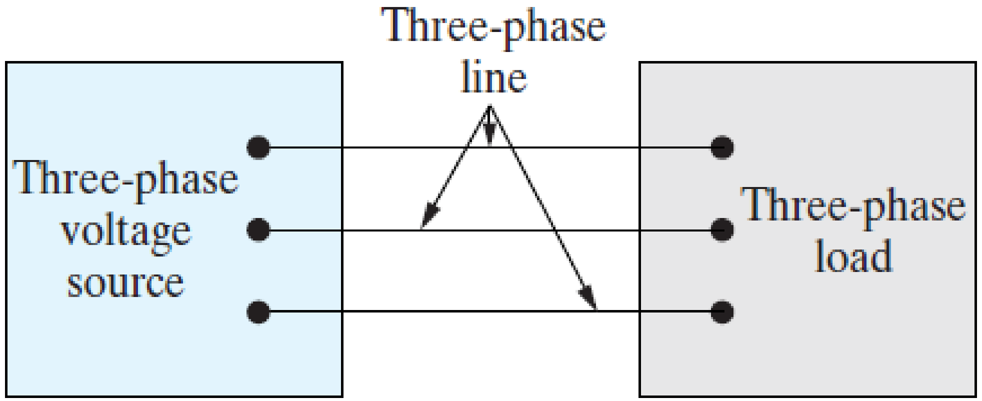


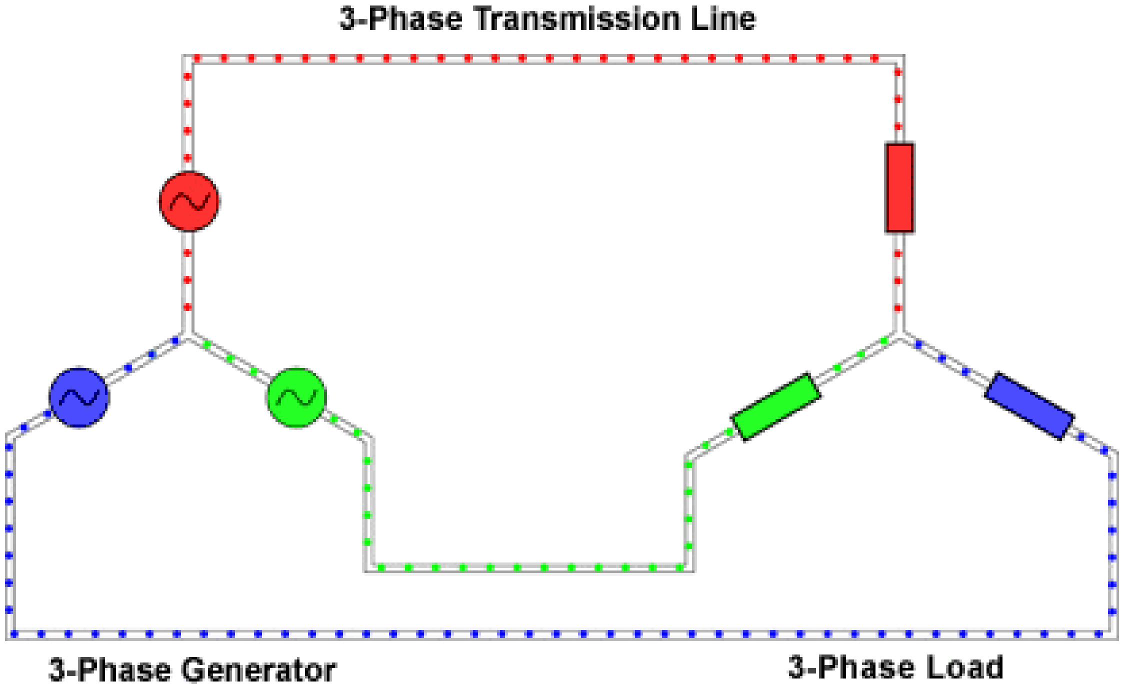
3-Phase Electricity Generation



More efficient in several ways compared to single-phase including transmission economy and constancy of power

3-Phase: Source vs Load



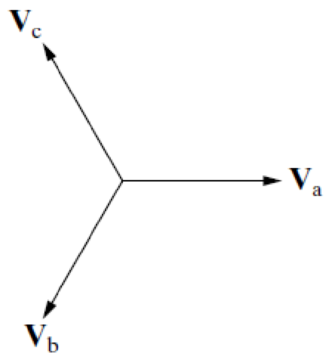


3-Phase: Balanced vs Imbalanced

- Balanced 3-Phase is a set of three AC voltages that satisfy following conditions:
  - Equal maximum amplitudes V_m
  - Equal frequencies ω
  - Each voltage exactly 120^0 out of phase with the other
- Imbalanced
  - When any of the three conditions above fails

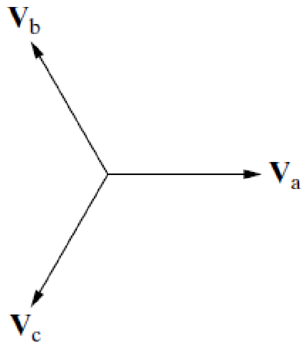
$$\mathbf{V_1} = V_m \angle \phi_a$$
$$\mathbf{V_2} = V_m \angle (\phi_a - 120^0)$$
$$\mathbf{V_3} = V_m \angle (\phi_a + 120^0)$$

Balanced 3-Phase: only two possibilities if $\phi_a = 0^0$



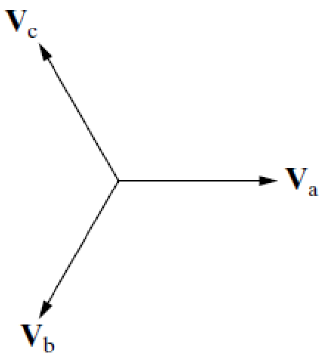
$$\mathbf{V}_a = V_m \angle 0^\circ,$$
$$\mathbf{V}_b = V_m \angle -120^\circ,$$
$$\mathbf{V}_c = V_m \angle +120^\circ,$$

Why 120^0? Hint: $\frac{360^0}{3}$



$$\mathbf{V}_a = V_m \angle 0^\circ,$$
$$\mathbf{V}_b = V_m \angle +120^\circ,$$
$$\mathbf{V}_c = V_m \angle -120^\circ.$$

Balanced 3-Phase: sum always zero



$$\mathbf{V}_a = V_m \angle 0^\circ,$$

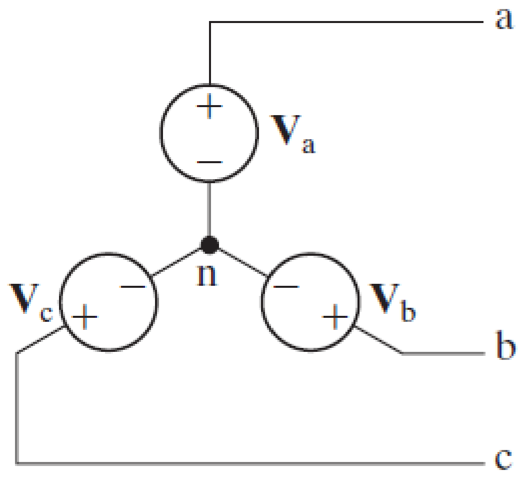
$$\mathbf{V}_b = V_m \angle -120^\circ,$$

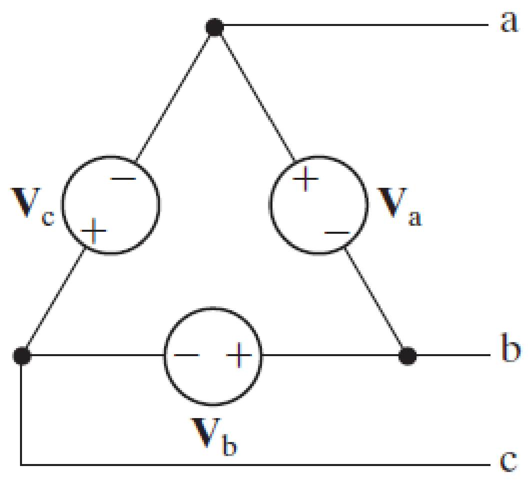
$$\mathbf{V}_c = V_m \angle +120^\circ,$$

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0.$$

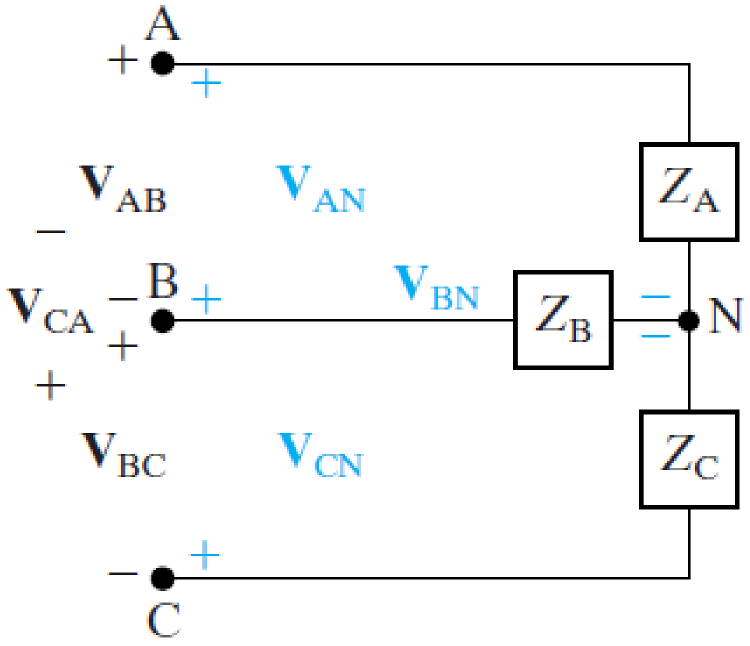
Why?
Check algebraically and graphically!

3-Phase: Wye vs Delta

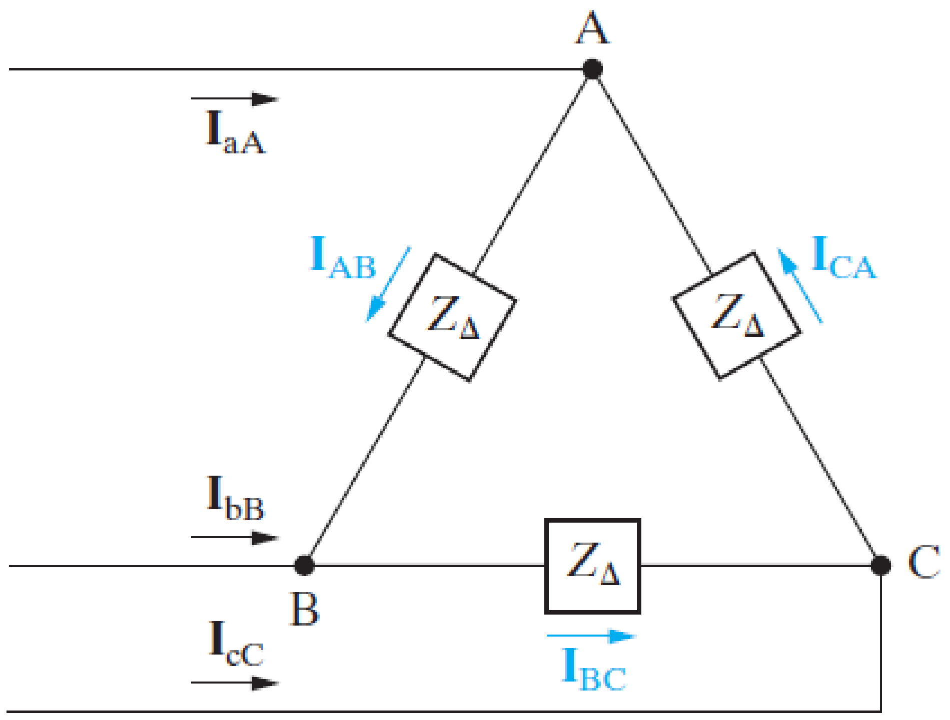




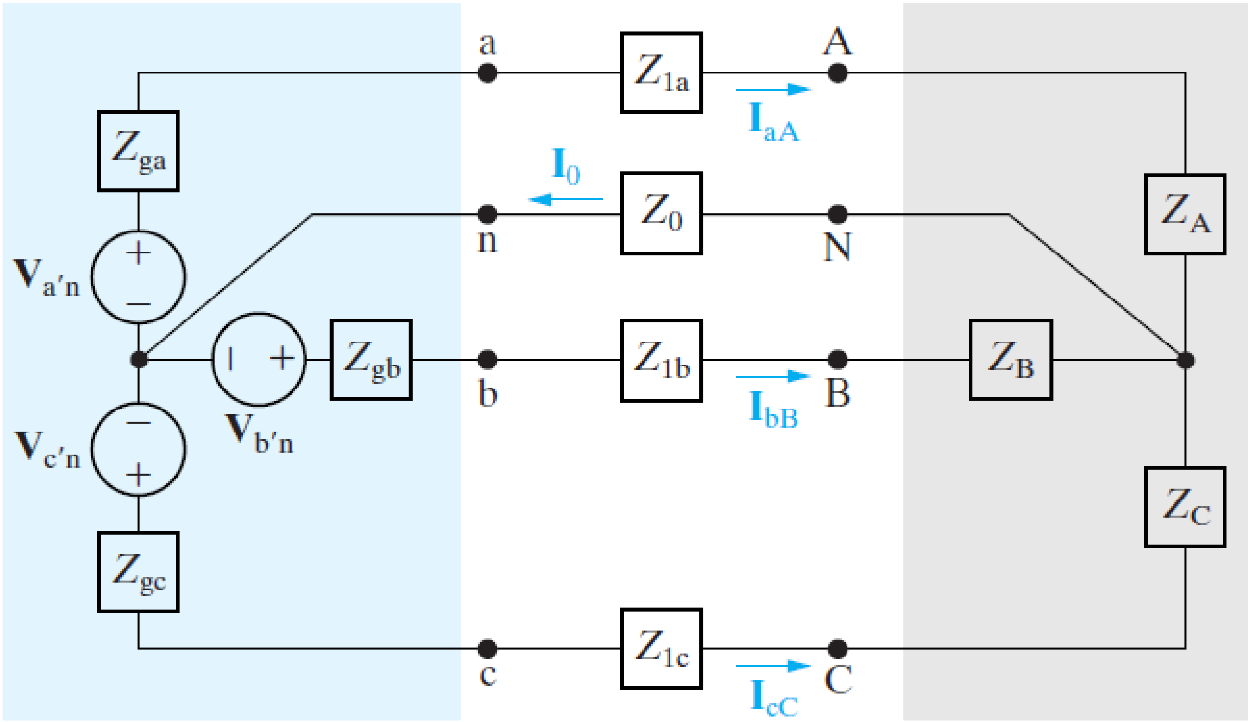
Wye: Line Voltage vs Phase Voltage



Delta: Line Current vs Phase Current



Balanced 3-Phase Wye-Wye Circuit



The entire circuit is balanced when the following four conditions are met

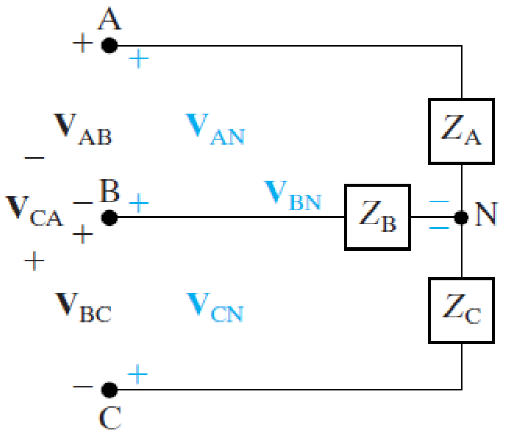
$\mathbf{V}_{a'n} + \mathbf{V}_{b'n} + \mathbf{V}_{c'n} = 0$ *Balanced source*

$Z_{ga} = Z_{gb} = Z_{gc}.$ *Equal source impedances*

$Z_{1a} = Z_{1b} = Z_{1c}.$ *Equal line impedances*

$Z_A = Z_B = Z_C.$ *Equal load impedances*

Wye-Wye: Voltages (load end)



Phase Voltages

$$\mathbf{V}_{AN} = V_\phi \angle 0°,$$

$$\mathbf{V}_{BN} = V_\phi \angle -120°,$$

$$\mathbf{V}_{CN} = V_\phi \angle +120°,$$

$$\mathbf{V}_{AN} = V_\phi \angle \phi_a$$
$$\mathbf{V}_{BN} = V_\phi \angle(\phi_a - 120^0)$$
$$\mathbf{V}_{CN} = V_\phi \angle(\phi_a + 120^0)$$

Line Voltages

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}, \quad = \sqrt{3} V_\phi \angle 30°,$$

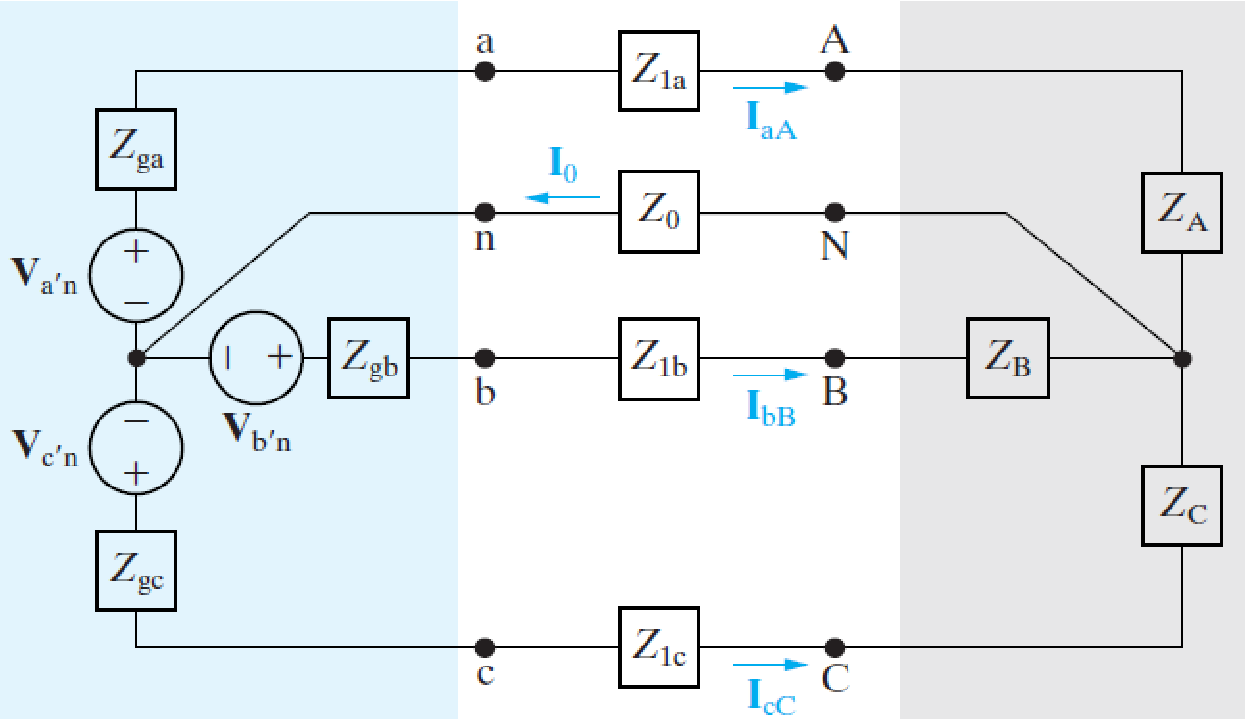
$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN}, \quad = \sqrt{3} V_\phi \angle -90°,$$

$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN}. \quad = \sqrt{3} V_\phi \angle 150°.$$

In general then:

$$\mathbf{V_{AB}} = (\sqrt{3} \angle 30^0 \,)\mathbf{V_{AN}}$$

Wye-Wye: Currents



$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{a'n}}{Z_\phi},$$

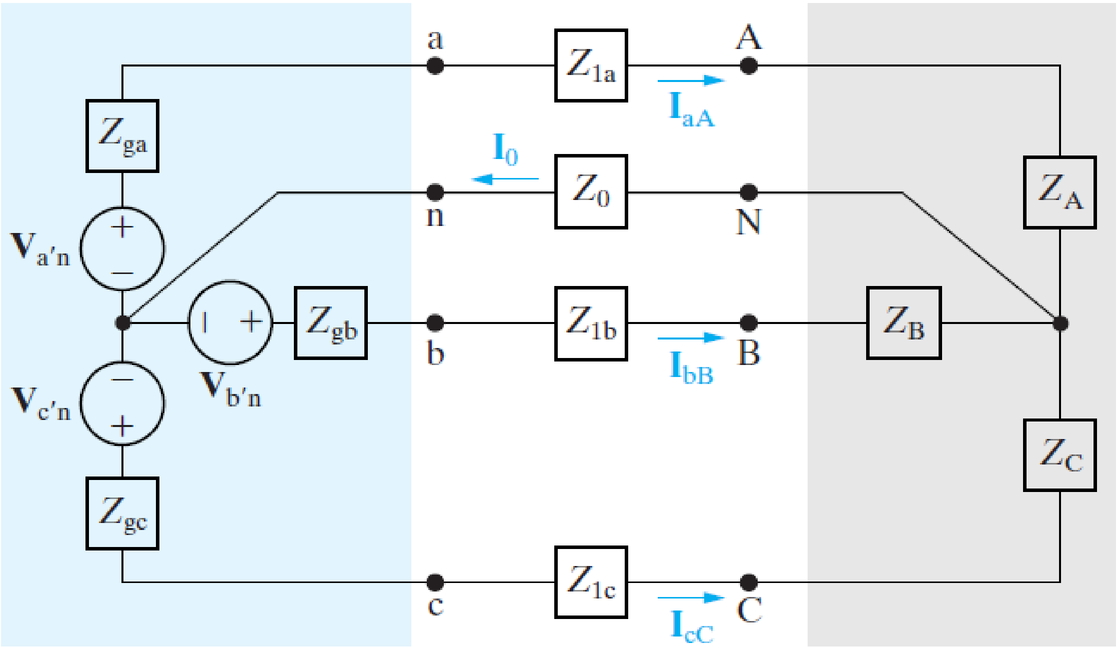
$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{b'n}}{Z_\phi},$$

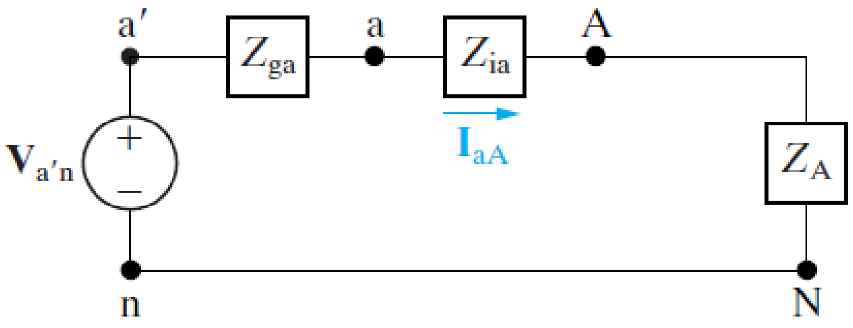
$$\mathbf{I}_{cC} = \frac{\mathbf{V}_{c'n}}{Z_\phi}.$$

In Wye-Wye, line current and phase current are the same thing

$$Z_\phi = Z_A + Z_{1a} + Z_{ga} = Z_B + Z_{1b} + Z_{gb} = Z_C + Z_{1c} + Z_{gc}.$$

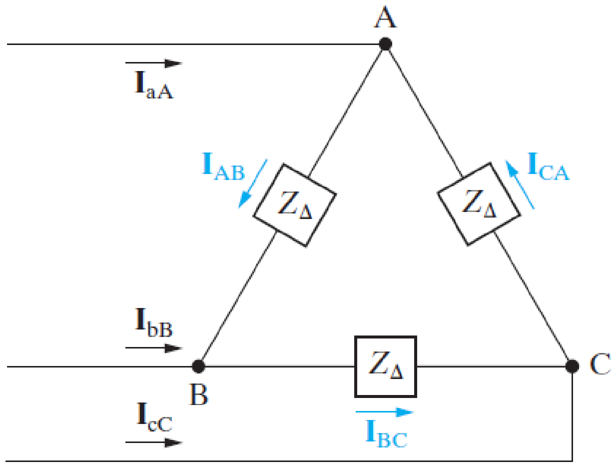
Single-Phase Equivalent Circuit





1. For analysis, we often extract a single-phase circuit from the 3-phase.
2. Once we have found the parameters of this single-phase, parameters of other two phases can be deduced from them.

Wye-Delta: Currents (load end)



Phase Currents

$$\mathbf{I}_{AB} = I_\phi \angle 0^\circ,$$

$$\mathbf{I}_{BC} = I_\phi \angle -120^\circ,$$

$$\mathbf{I}_{CA} = I_\phi \angle 120^\circ.$$

Line Currents

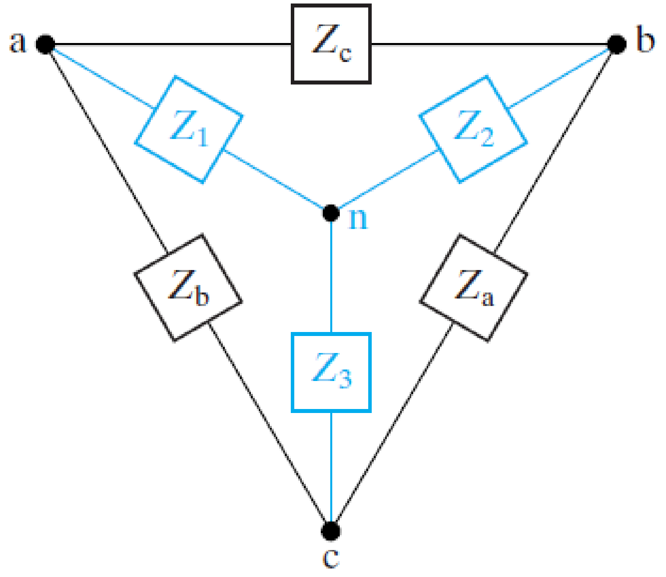
$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \sqrt{3} I_\phi \angle -30^\circ,$$

$$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \sqrt{3} I_\phi \angle -150^\circ,$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = \sqrt{3} I_\phi \angle 90^\circ.$$

$$\boldsymbol{I_{AB}} = I_\phi \angle \phi_a$$
$$\boldsymbol{I_{BC}} = I_\phi \angle (\phi_a - 120^0)$$
$$\boldsymbol{I_{CA}} = I_\phi \angle (\phi_a + 120^0)$$

Transformation: Delta → Wye



For balanced case, we have:

$$Z_Y = \frac{Z_\Delta}{3},$$

Where:

$$Z_Y = Z_1 = Z_2 = Z_3$$
$$Z_\Delta = Z_a = Z_b = Z_c$$

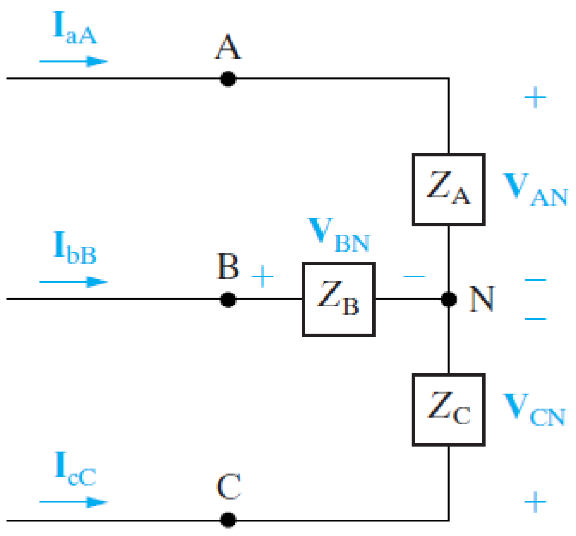
Practice Examples

Average, Reactive, and Complex Power for 3-phase Circuits

Notes:

1. All the given power formulas assume balanced 3-phase circuits
2. All the given power formulas assume effective (rms) values of current and voltage

Balanced Wye : Average (real) Power



$$P_A = |\mathbf{V}_{AN}||\mathbf{I}_{aA}| \cos(\theta_{vA} - \theta_{iA}),$$

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi,$$

Same average power in each phase

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi.$$

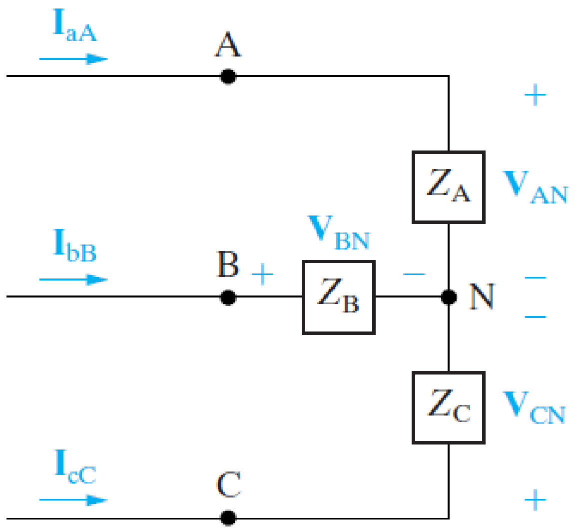
Total Average Power

$$V_\phi = |\mathbf{V}_{AN}| = |\mathbf{V}_{BN}| = |\mathbf{V}_{CN}|,$$

$$I_\phi = |\mathbf{I}_{aA}| = |\mathbf{I}_{bB}| = |\mathbf{I}_{cC}|,$$

$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}.$$

Balanced Wye : Average (real) Power

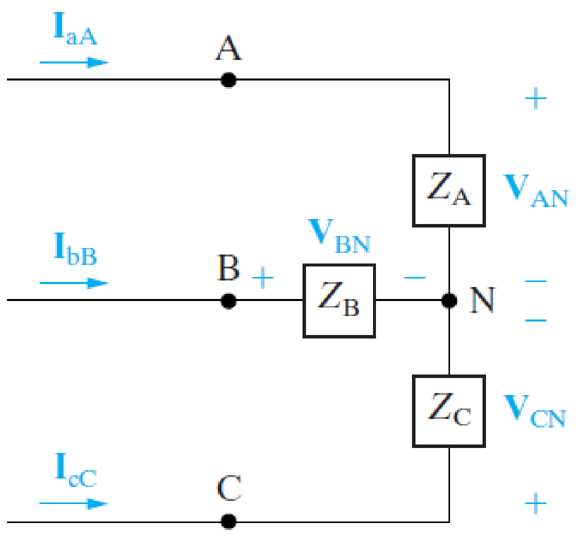


$$P_T = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\theta_\phi$$

$$= \sqrt{3} V_L I_L \cos\theta_\phi.$$

Total Average Power in terms of line parameters

Balanced Wye : Reactive Power



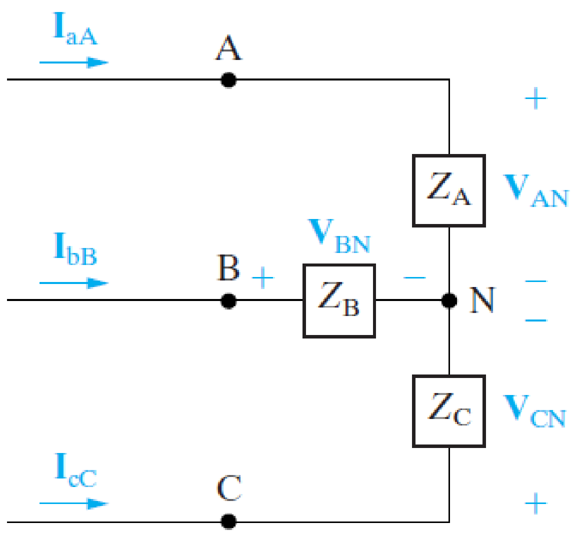
$$Q_\phi = V_\phi I_\phi \sin\theta_\phi,$$

Same reactive power in each phase

$$Q_T = 3Q_\phi = \sqrt{3}V_L I_L \sin\theta_\phi.$$

Total Reactive Power

Balanced Wye : Complex Power



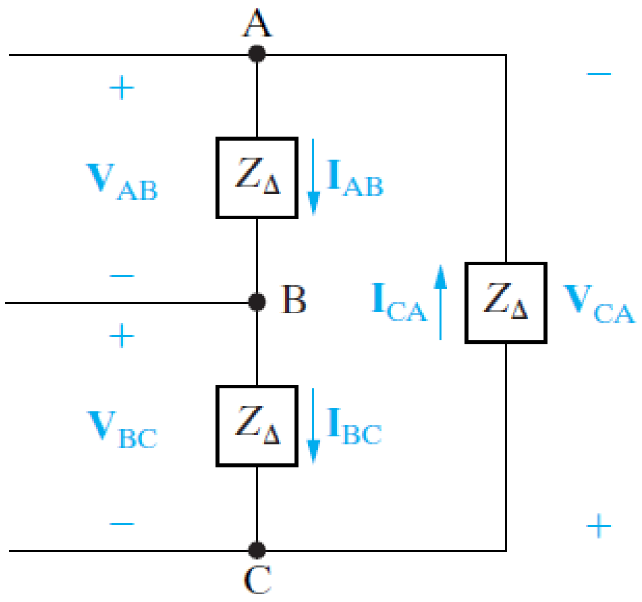
$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi \mathbf{I}_\phi^*$$

Same complex power in each phase

$$S_T = 3S_\phi = \sqrt{3} V_L I_L \angle\theta_\phi^\circ .$$

Total Complex Power

Balanced Delta : Power equations same as for Wye



Only difference now are the definitions of phase voltage, phase current, and power factor angle, as follows:

$$|\mathbf{V}_{AB}| = |\mathbf{V}_{BC}| = |\mathbf{V}_{CA}| = V_{\phi},$$

$$|\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}| = I_{\phi},$$

$$\theta_{vAB} - \theta_{iAB} = \theta_{vBC} - \theta_{iBC} = \theta_{vCA} - \theta_{iCA} = \theta_{\phi},$$

Practice Examples

Questions?? Thoughts??

EE 202
Electric Circuit Analysis

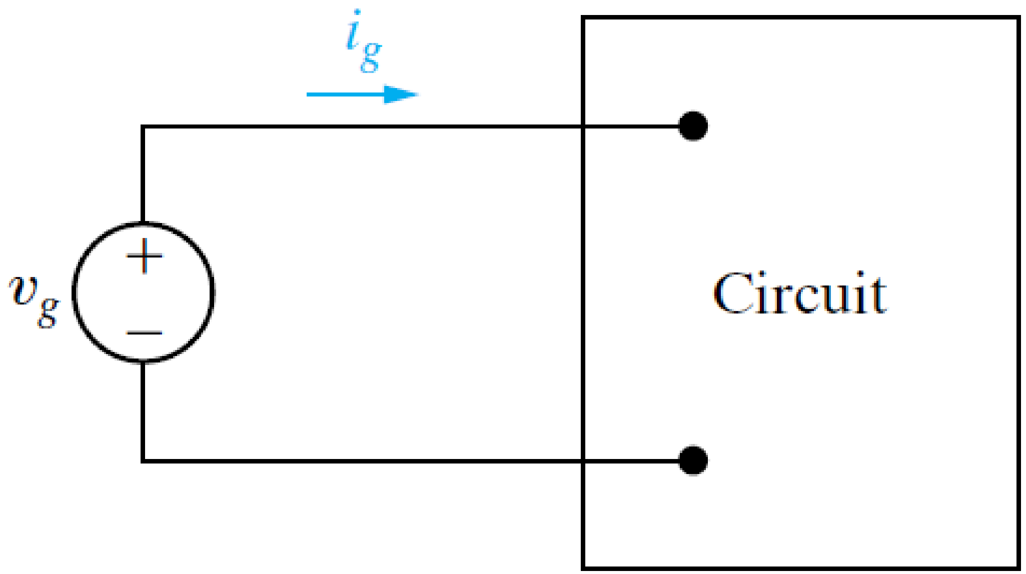
with

Dr. Naveed R. Butt

@

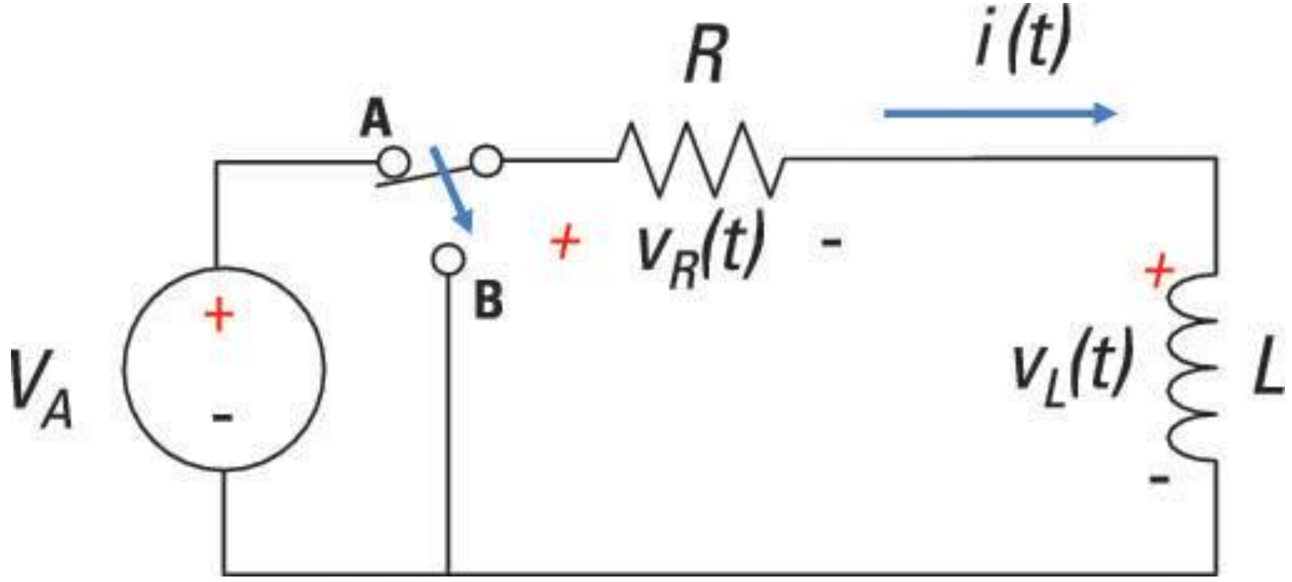
Jouf University

Circuit Analysis – mostly about v, i, p



$$p = vi$$

What happens when circuit configuration changes?

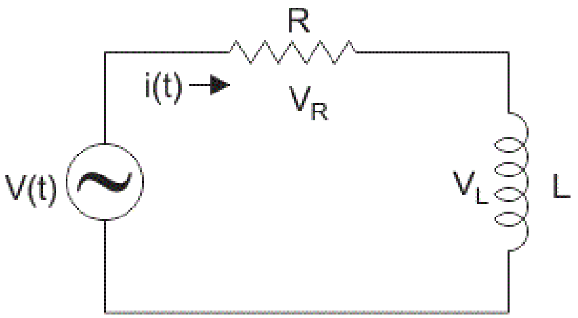


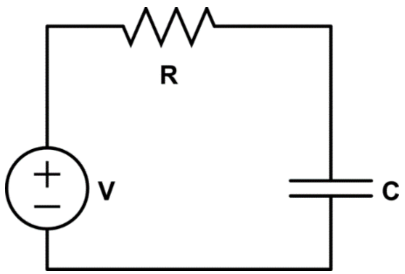
$$i_L(t) = I_\infty + (I_o - I_\infty)e^{-t/\tau}$$

Circuits: First-Order vs Second-Order

First-Order

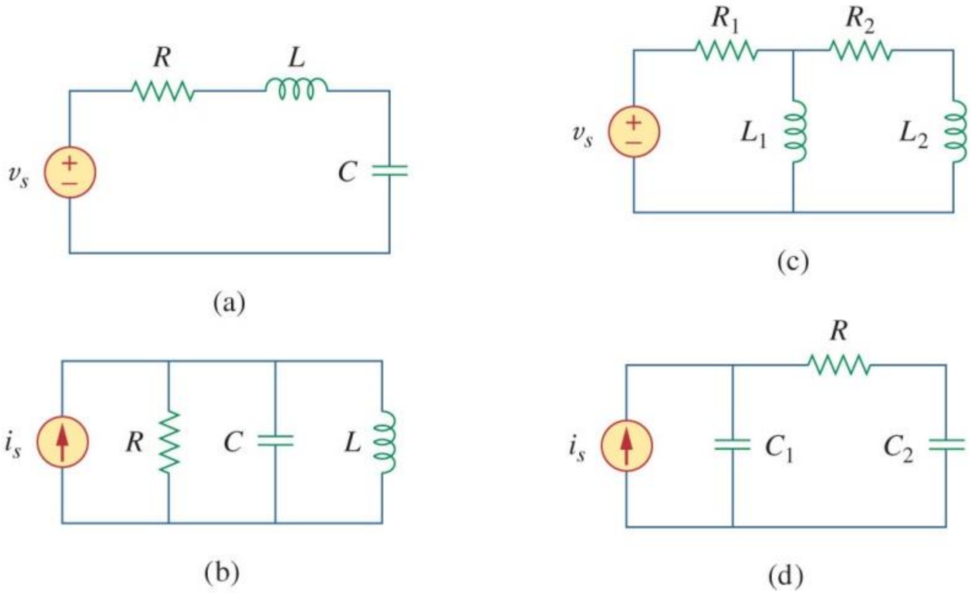
- Only one storage element (one inductor or one capacitor) – e.g., RL, RC
- Mathematically described by first-order differential equations



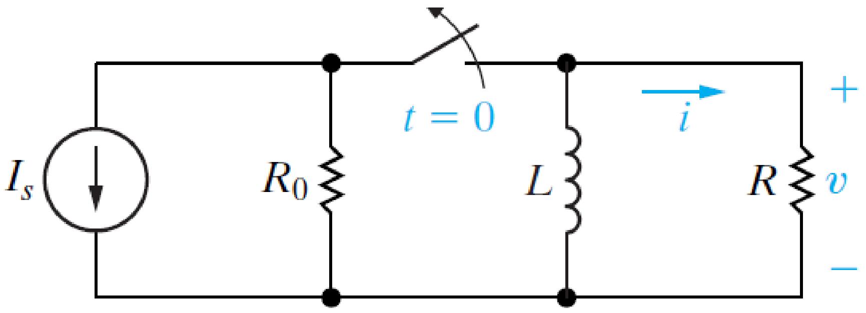


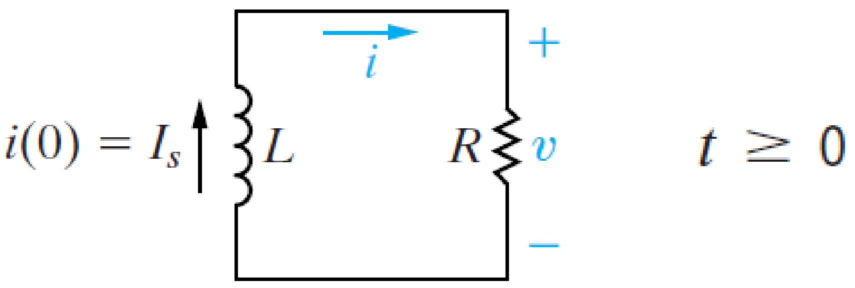
Second-Order

- Two storage elements (Inductors or Capacitors or mix) – e.g., RLC
- Mathematically described by second-order differential equations



Response: Natural vs Step

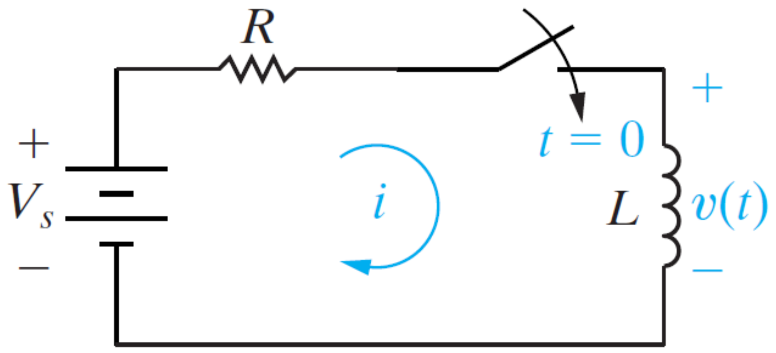




Natural Response

- Behavior of a circuit when no voltage (or current) source is present or is suddenly removed at $t = 0$
- Circuit basically driven by initial conditions

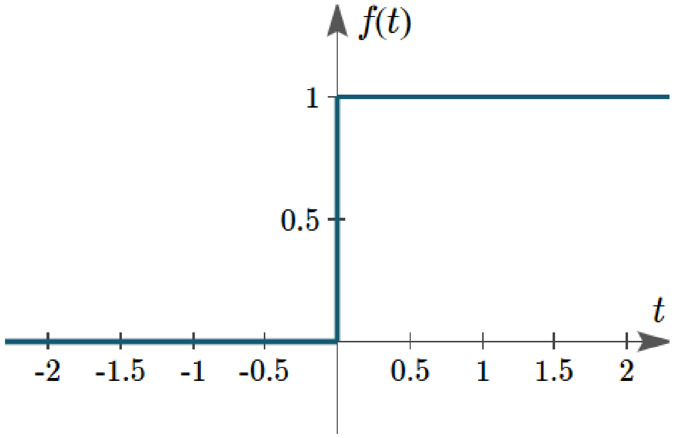
Response: Natural vs Step



Step Response

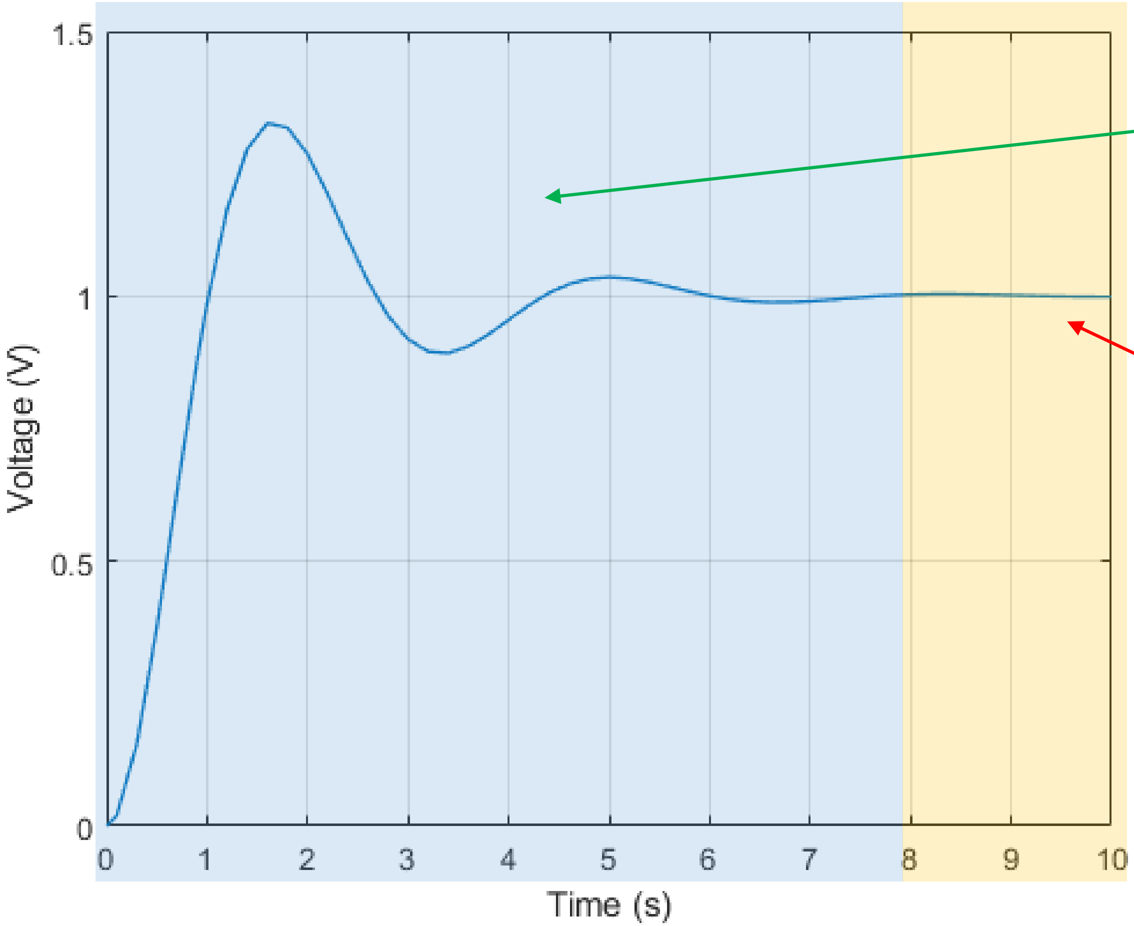
- Behavior of a circuit when a fixed voltage (or current) is applied from $t = 0$

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



Graph of $f(t) = u(t)$, the unit step function.

Response: Transient vs Steady-State



Transient Response

- Response immediately after the voltage (or current) source is applied

Steady-State Response

- Response once the circuit has reached equilibrium
- Typically a reasonable amount of time after source applied

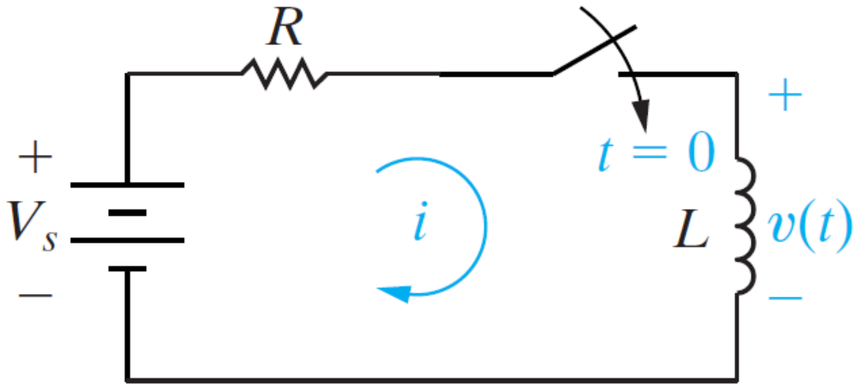
Four Important Parameters to Consider

Source Value

- How much voltage (or current) is applied to the circuit at $t = 0$?
- Denoted : V_s or I_s

Initial Condition

- How much voltage (or current) was already there in the circuit just before switching?
- Denoted : V_0 or I_0



$$i(0^-) = i(0^+) = I_0,$$

Four Important Parameters to Consider

Steady-State Value

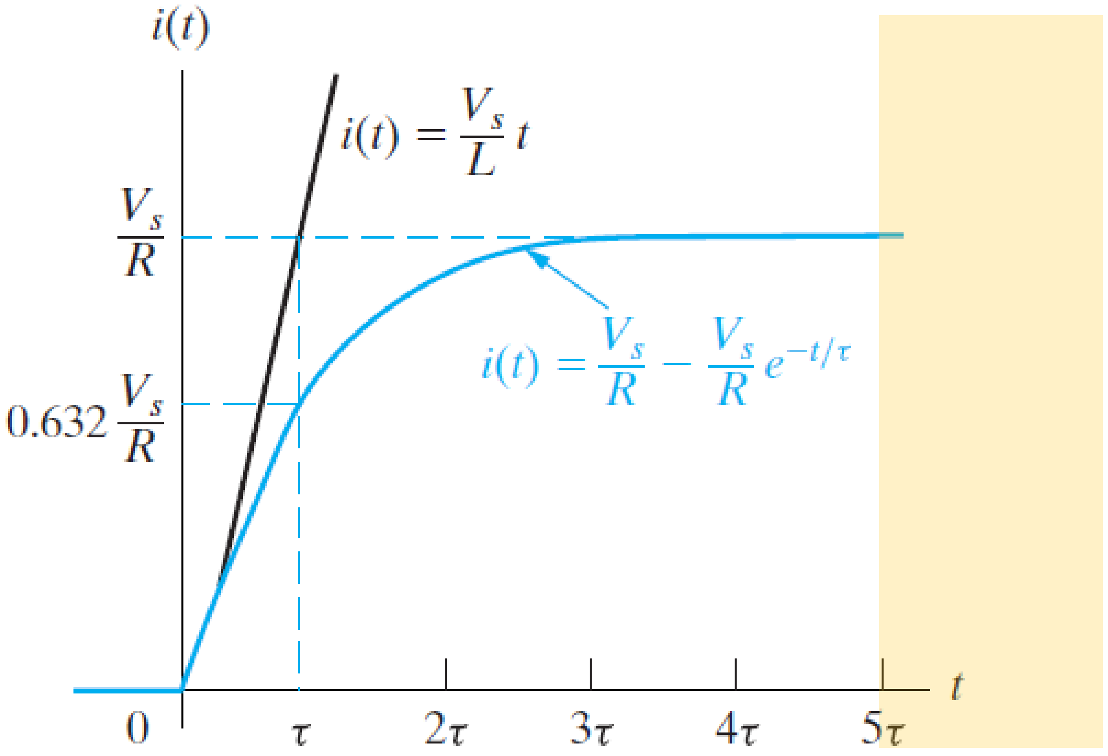
- How much voltage (or current) is there in the steady-state (i.e. when a long time has passed)

- Denoted **:** V_∞ or I_∞

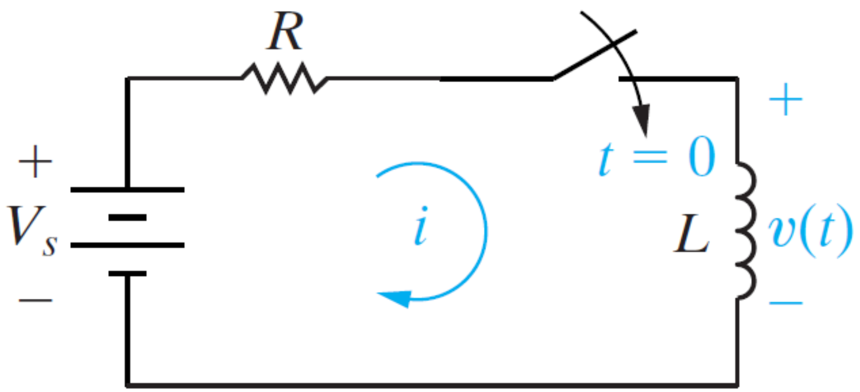
Four Important Parameters to Consider

Time Constant

- A parameter used to find how quickly the circuit goes towards its steady state
- Denoted : τ
- Typically steady-state defined as response after $t \geq 5\,\tau$



RL Step Response - Current



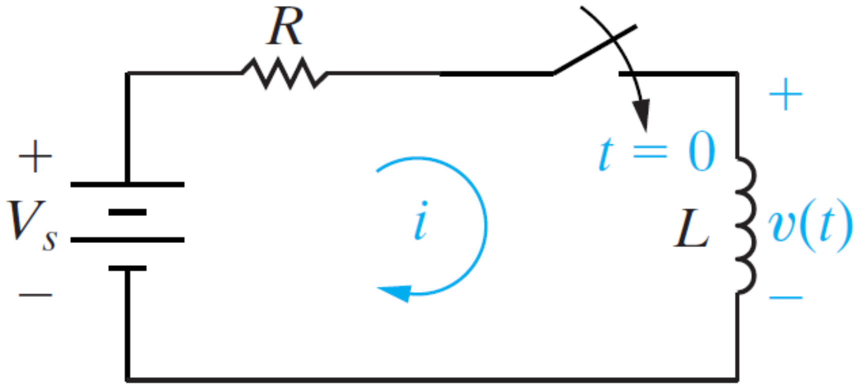
I_∞

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-(R/L)t}.$$

$$i(0^-) = i(0^+) = I_0,$$

Time Constant: $\tau = \frac{L}{R}$

RL Step Response - Voltage



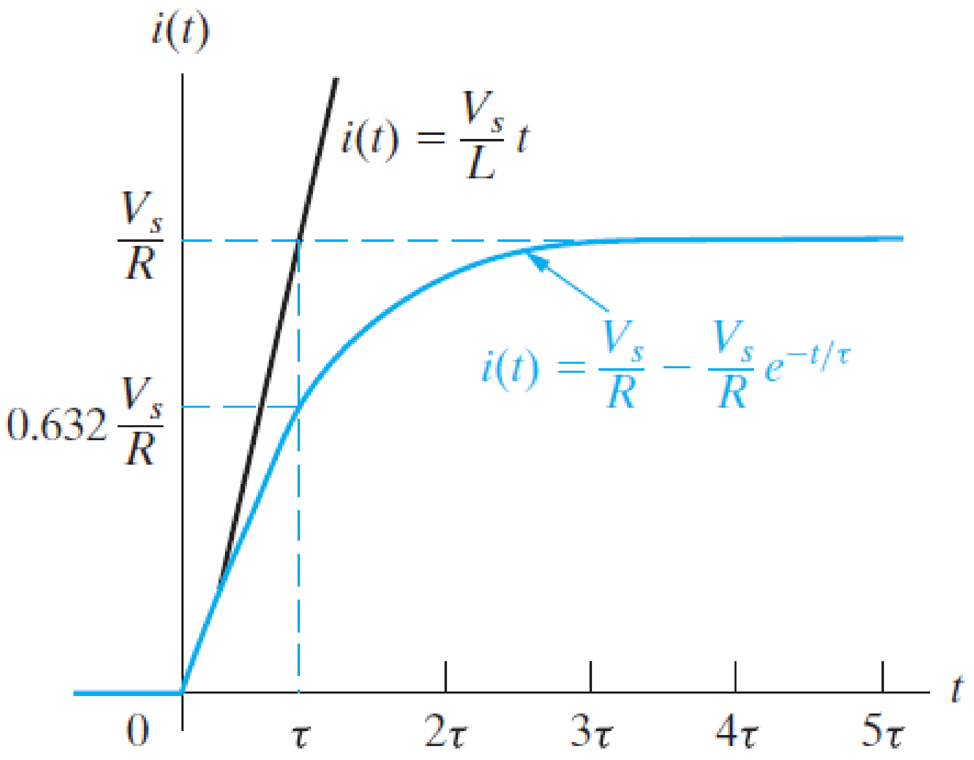
$$v = (V_s - I_0 R)e^{-(R/L)t}.$$

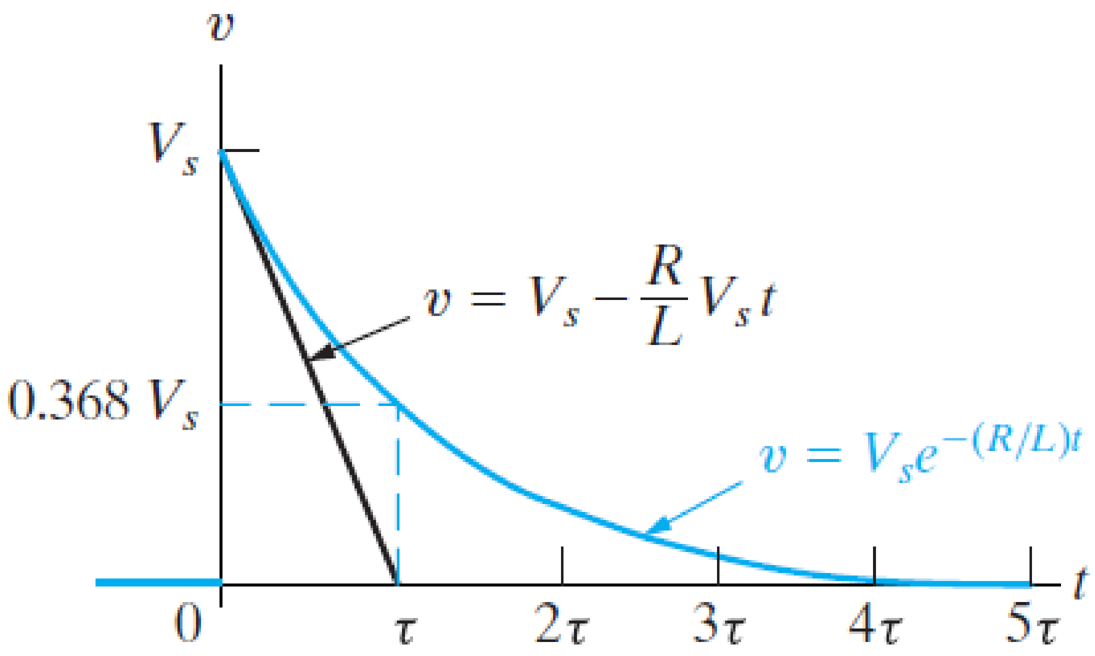
$$i(0^-) = i(0^+) = I_0,$$

Time Constant: $\tau = \frac{L}{R}$

RL Step Response

Case: $I_0 = 0$

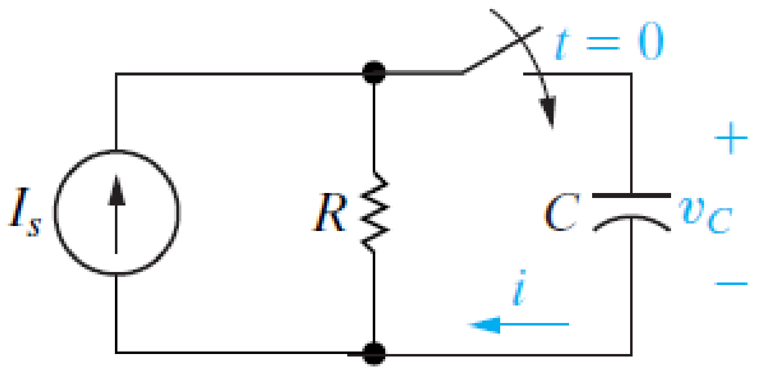




$$i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-(R/L)t}.$$

$$v = V_s e^{-(R/L)t}.$$

RC Step Response - Voltage



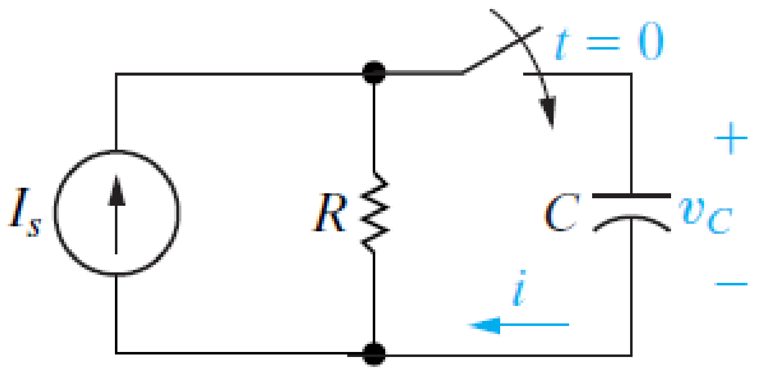
$$v(0^-) = v(0) = v(0^+) = V_0,$$

V_∞

$$v_C = I_s R + (V_0 - I_s R)e^{-t/RC}, \quad t \geq 0.$$

Time Constant: $\tau = RC$

RC Step Response - Current



$$i = \left(I_s - \frac{V_0}{R}\right)e^{-t/RC}, \qquad t \geq 0^+,$$

$$v(0^-) = v(0) = v(0^+) = V_0,$$

Time Constant: $\tau = RC$

General Formulas (Natural & Step Response)

Current through inductor in standard RL formation

Time constant: $\tau = \frac{L}{R}$

$$i_L(t) = I_\infty + (I_o - I_\infty)e^{-t/\tau}$$

Steady-state current through inductor

Initial current through inductor

General Formulas (Natural & Step Response)

Voltage across capacitor in standard RC formation

Time constant: $\tau = RC$

$$v_C(t) = V_\infty + (V_o - V_\infty)e^{-t/\tau}$$

Steady-state voltage across capacitor

Initial voltage across capacitor

Practice Examples

Questions?? Thoughts??

EE 202
Electric Circuit Analysis

with

Dr. Naveed R. Butt

@

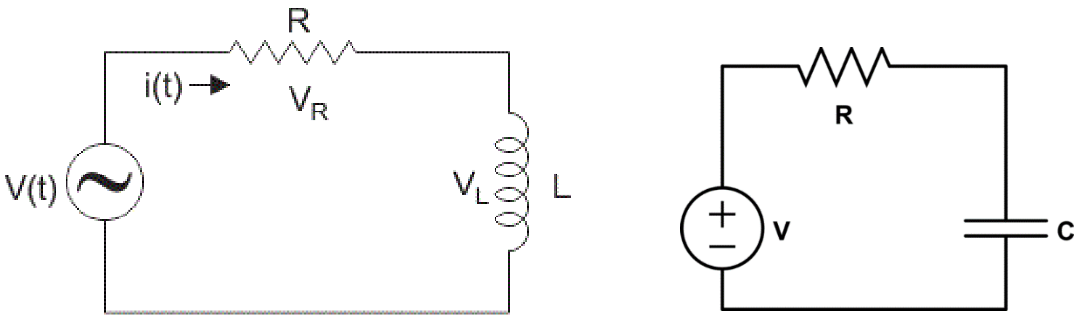
Jouf University

Circuits: First-Order vs Second-Order

First-Order

- Only one storage element (one inductor or one capacitor) – e.g., RL, RC

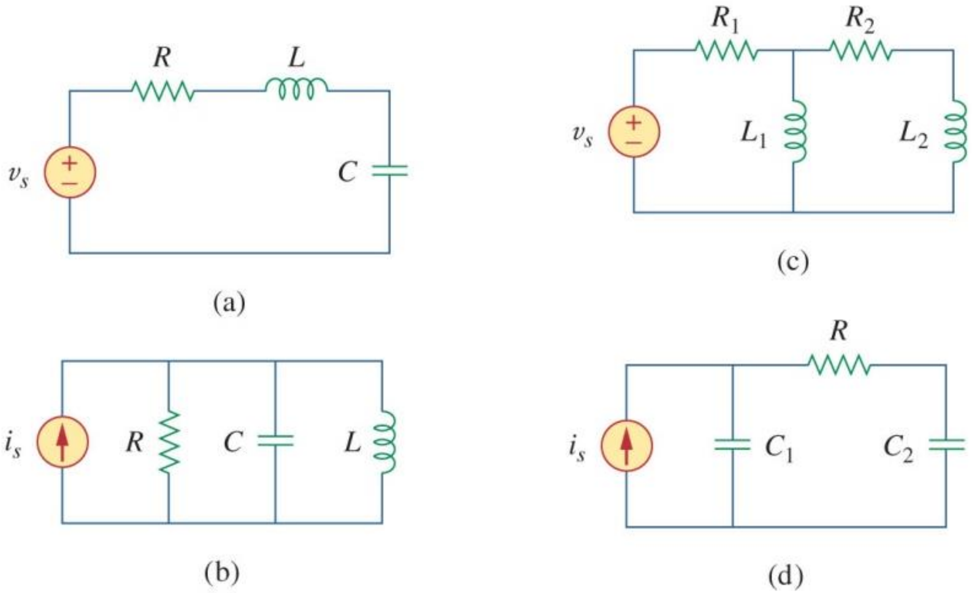
- Mathematically described by first-order differential equations



Second-Order

- Two storage elements (Inductors or Capacitors or mix) – e.g., RLC

- Mathematically described by second-order differential equations



Problem – differential equations are hard!!

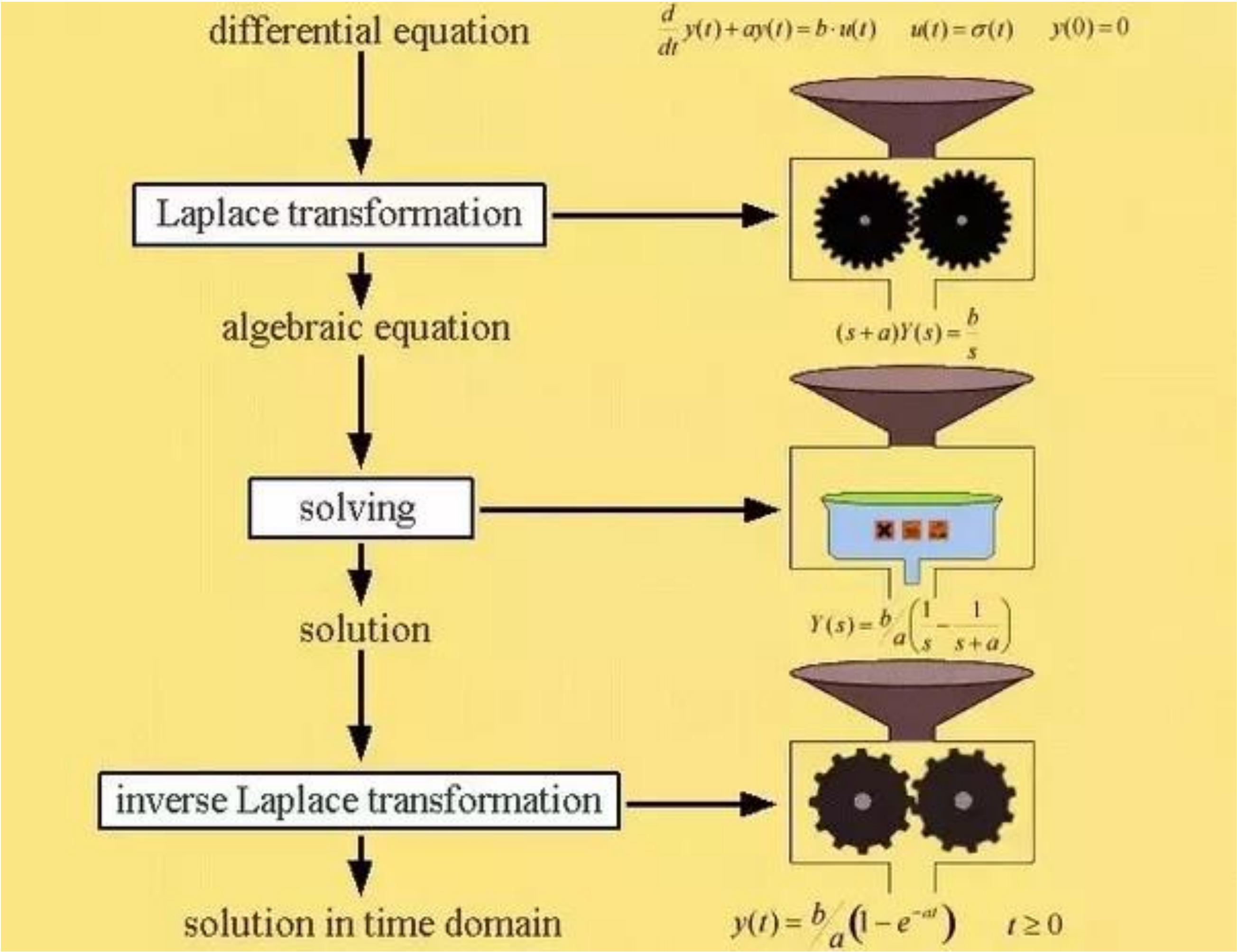
We have a problem!!

- Higher-order circuits lead to higher-order differential equations
- To analyze such circuits we need to solve the higher-order differential equations
- But higher-order differential equations are generally hard to solve!!

Solution:

Laplace

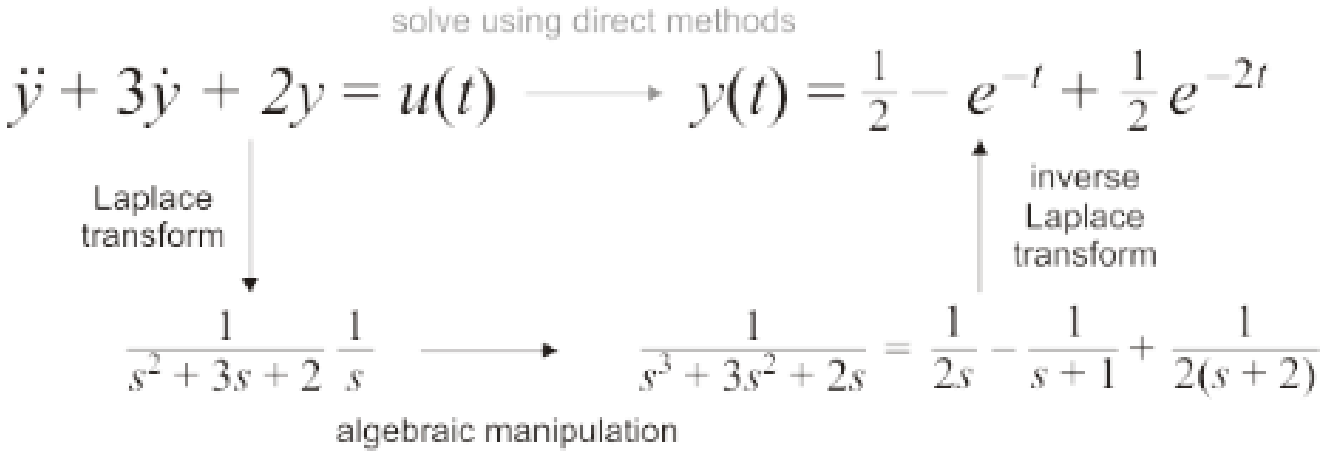
Laplace Transform – why?



Key

Laplace Transform converts differential equations (derivatives and integrals) into simple algebraic equations!!

Laplace Transform: Calculus → Algebra



Key

Laplace Transform converts differential equations (derivatives and integrals) into simple algebraic equations!!

Type	$f(t)$ $(t > 0-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Key

Formula tables exist for convenient use of Laplace Transform

Notation

$$\mathscr{L}\{f(t)\} = F(s)$$

Laplace Transforms of Derivatives and Integrals

$$\mathscr{L}\{f(t)\} = F(s)$$

Time-Domain	*s*-Domain (Laplace)
$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
$\int_0^t f(x)\,dx$	$\frac{F(s)}{s}$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) -$ (initial conditions)

Using Laplace – a simple example

Given: $\frac{df(t)}{dt} + 5f(t) = 0$ And $f(0^-) = 2$

Find: $f(t)$

Solution:

$$sF(s) - f(0^-) + 5F(s) = 0$$

$$sF(s) - 2 + 5F(s) = 0$$

$$sF(s) + 5F(s) = 2$$

$$(s + 5)F(s) = 2$$

$$F(s) = \frac{2}{s + 5}$$

$$f(t) = 2e^{-5t}$$

Laplace Formulas we use:

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-)$$

$$\mathcal{L}\{e^{-at}\} = \frac{1}{s + a}$$

Using Laplace – a simple example

Given: $2\frac{df(t)}{dt} + f(t) = 0$ And $f(0^-) = 3$

Find: $f(t)$

Solution:

$$2(sF(s) - f(0^-)) + F(s) = 0$$

$$2(sF(s) - 3) + F(s) = 0$$

$$2sF(s) - 6 + F(s) = 0$$

$$2sF(s) + F(s) = 6$$

$$(2s + 1)F(s) = 6$$

$$F(s) = \frac{6}{2s+1} = \frac{6}{\frac{2}{2}(2s+1)} = \frac{1}{2} \times \frac{6}{s + \frac{1}{2}} = \frac{1}{2} \times 6 \times \frac{1}{s + 1/2}$$

$$f(t) = \frac{1}{2} \times 6 \times e^{-\frac{1}{2}t} = 3e^{-\frac{1}{2}t}$$

Laplace Formulas we use:

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-)$$

$$\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$$

Laplace for Circuit Analysis

- Electrical Quantities in Laplace
  - Voltage
  - Current
- Basic Elements in Laplace
  - Resistor
  - Inductor
  - Capacitor
  - DC source connected at $t = 0$
  - AC source connected at $t = 0$
- Some Second-Order Circuits in Laplace
  - RLC with DC source
  - RLC with AC source

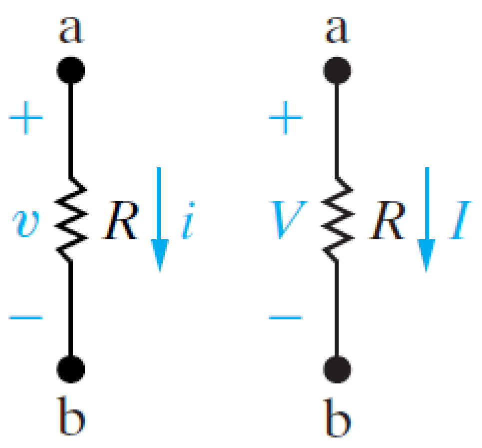
Laplace – Voltage and Current

$$V = \mathscr{L}\{v\} \quad \text{and} \quad I = \mathscr{L}\{i\}$$

Unit: ***volt-seconds***

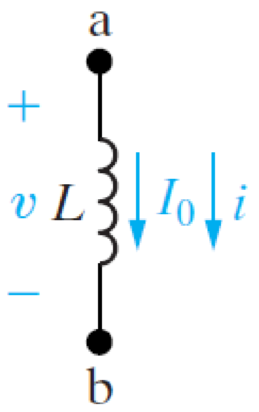
Unit: ***ampere-seconds***

Laplace – Resistor



Time	Phasor	Laplace
$v = Ri.$	$\mathbf{V} = R\mathbf{I},$	$V = RI$

Laplace – Inductor (with initial current I_0)



Time

$$v = L\frac{di}{dt}.$$

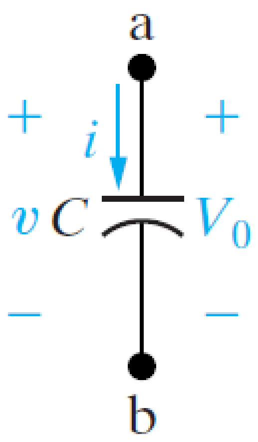
Phasor

$$\mathbf{V} = j\omega L\mathbf{I}.$$

Laplace

$$V = sLI - LI_0$$

Laplace – Capacitor (with initial voltage V_0)



Time	Phasor	Laplace
$i = C\frac{dv}{dt}.$	$\mathbf{I} = j\omega C\mathbf{V}.$	$I = sCV - CV_0$

Laplace – Impedances (assuming $V_0 = 0, I_0 = 0$)

	Phasor	Laplace
Ohm's Law:	$\mathbf{V} = Z\mathbf{I},$	$V = ZI,$
Resistor:	$Z = R$	$Z = R$
Inductor:	$Z = j\omega L$	$Z = sL$
Capacitor:	$Z = \dfrac{1}{j\omega C}$	$Z = \dfrac{1}{sC}$

Laplace - Sources

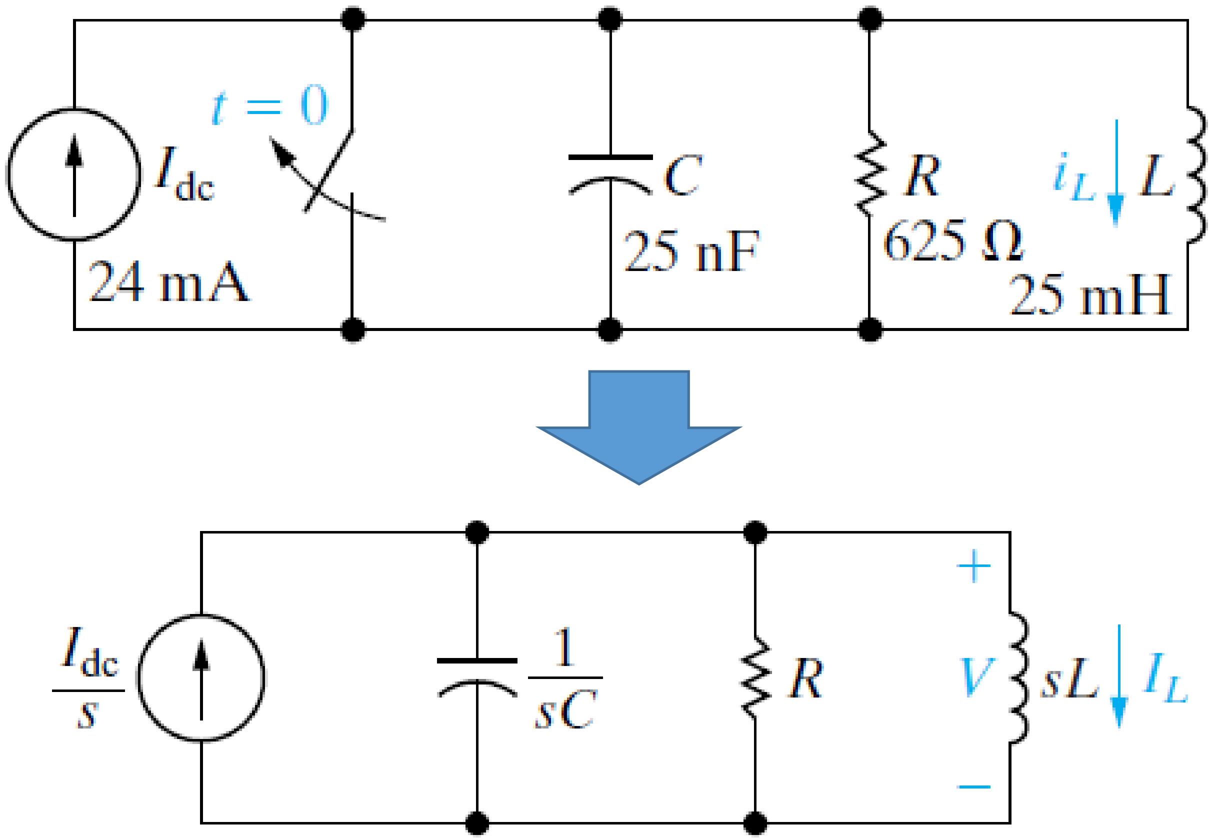
Laplace Form

DC Voltage Source (connected at $t = 0$)	$\frac{V_{dc}}{s}$
DC Current Source (connected at $t = 0$)	$\frac{I_{dc}}{s}$
AC Voltage Source (connected at $t = 0$) $v_{ac} = V_m \cos(\omega t)$	$\frac{sV_m}{s^2 + \omega^2}$
AC Current Source (connected at $t = 0$) $i_{ac} = I_m \cos(\omega t)$	$\frac{sI_m}{s^2 + \omega^2}$

Laplace – RLC Circuit with DC Source

Time

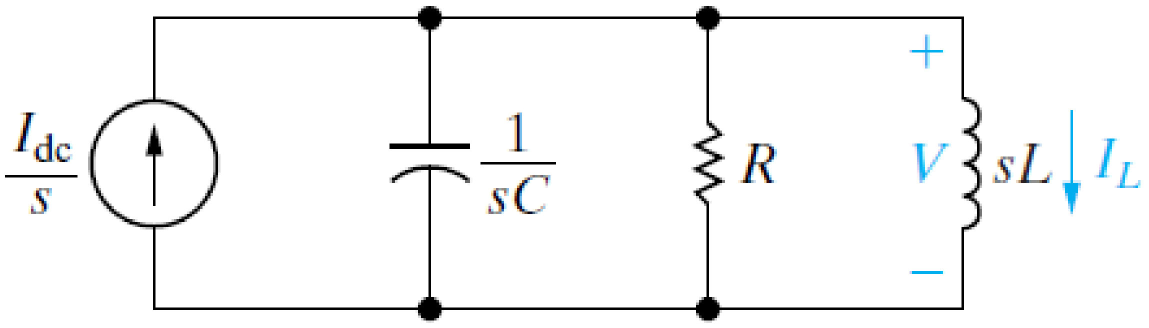
Laplace



Analysis Task: Find i_L

Assumptions:
- DC current source connected at $t = 0$
- All initial conditions zero

Laplace – RLC Circuit with DC Source



$$I_L = \frac{V}{sL}$$

$$sCV + \frac{V}{R} + \frac{V}{sL} = \frac{I_{dc}}{s}.$$

$$V = \frac{I_{dc}/C}{s^2 + (1/RC)s + (1/LC)}.$$

$$I_L = \frac{I_{dc}/LC}{s[s^2 + (1/RC)s + (1/LC)]}.$$

$$I_L = \frac{384 \times 10^5}{s(s^2 + 64{,}000s + 16 \times 10^8)}.$$

$$I_L = \frac{384 \times 10^5}{s(s + 32{,}000 - j24{,}000)(s + 32{,}000 + j24{,}000)}.$$

$$I_L = \frac{K_1}{s} + \frac{K_2}{s + 32{,}000 - j24{,}000} + \frac{K_2^*}{s + 32{,}000 + j24{,}000}.$$

Using "Partial Fraction Decomposition"

$$K_1 = \frac{384 \times 10^5}{16 \times 10^8} = 24 \times 10^{-3},$$

$$K_2 = \frac{384 \times 10^5}{(-32{,}000 + j24{,}000)(j48{,}000)} = 20 \times 10^{-3} \angle 126.87^\circ.$$

By Inverse Laplace

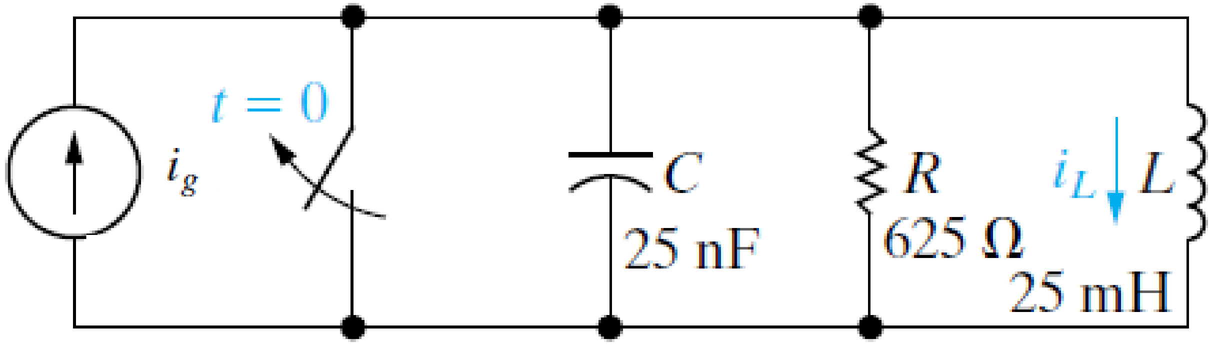
$$i_L = [24 + 40e^{-32{,}000t} \cos(24{,}000t + 126.87^\circ)]u(t)\text{mA}.$$

Laplace – RLC Circuit with AC Source

$i_g = I_m \cos \omega t$ A,

$I_m = 24$ mA and $\omega = 40{,}000$ rad/s.

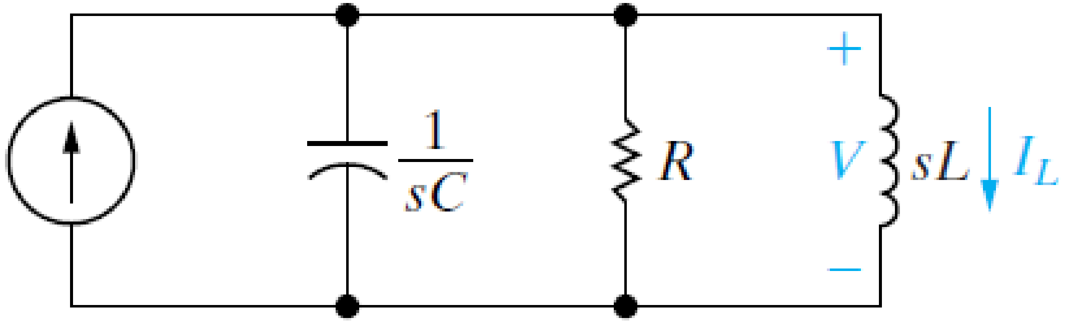
Time



Analysis Task: Find i_L

Laplace

$I_g = \dfrac{sI_m}{s^2 + \omega^2}.$

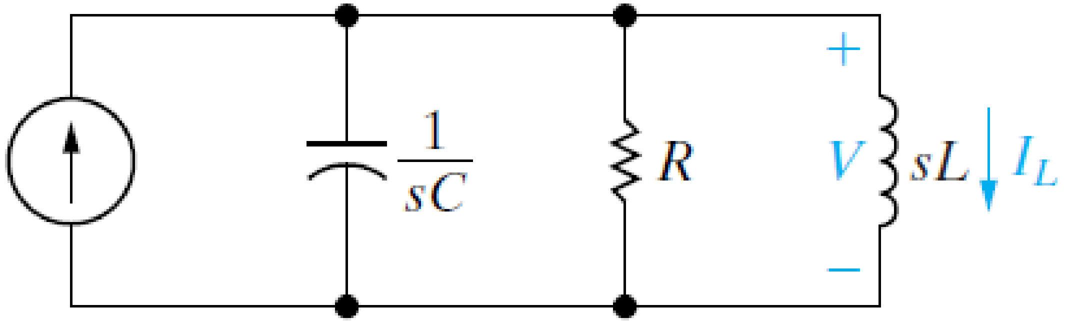


Assumptions:

- AC current source connected at $t = 0$
- All initial conditions zero

Laplace – RLC Circuit with AC Source

$$I_g = \frac{sI_m}{s^2 + \omega^2}.$$



$$I_L = \frac{V}{sL}$$

$$sCV + \frac{V}{R} + \frac{V}{sL} = \frac{sI_m}{s^2 + \omega^2}$$

Remaining steps similar to DC case …

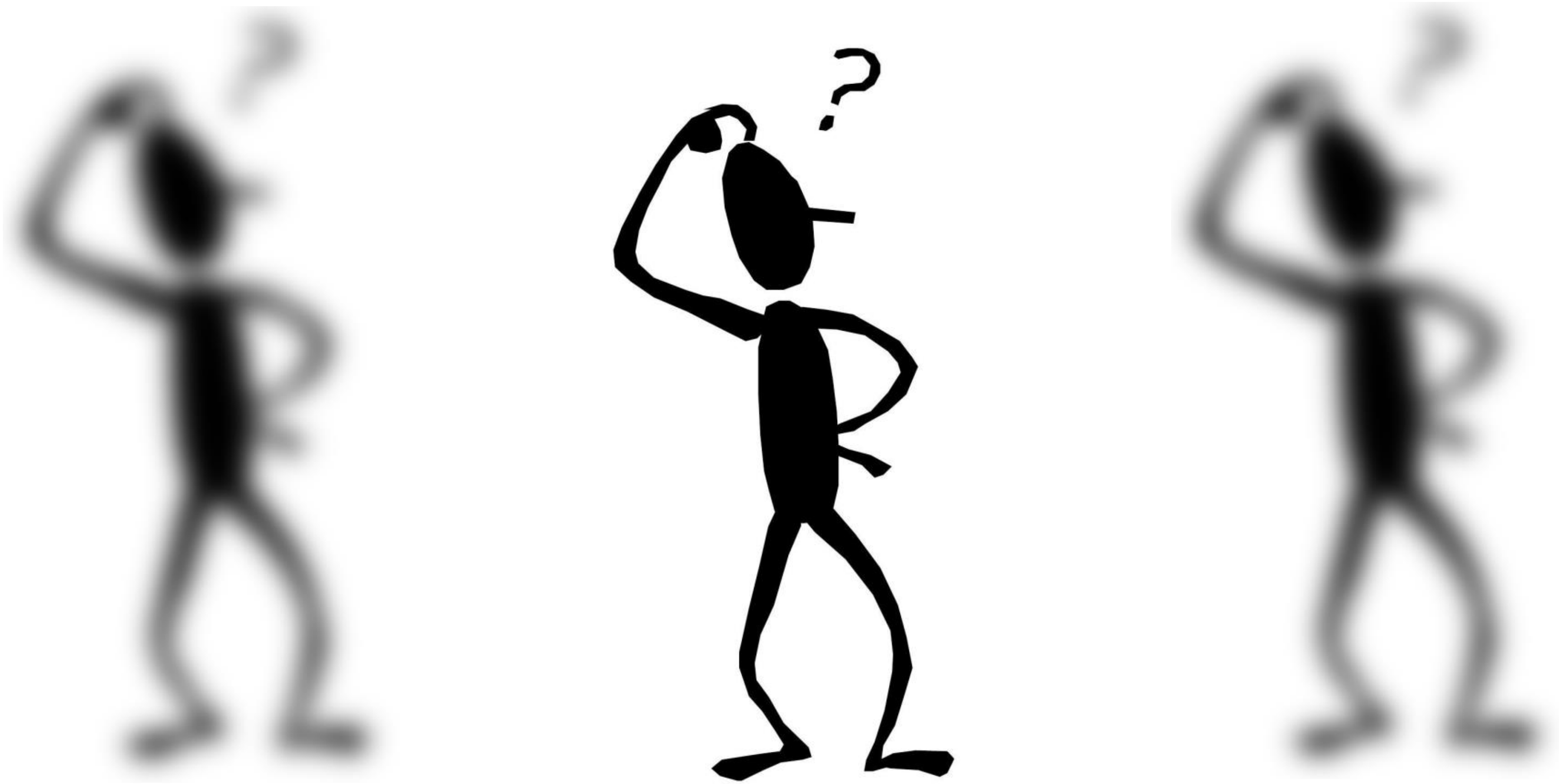
$$I_L = \frac{V}{sL} = \frac{(I_m/LC)s}{(s^2 + \omega^2)[s^2 + (1/RC)s + (1/LC)]}.$$

$$I_L = \frac{384 \times 10^5 s}{(s^2 + 16 \times 10^8)(s^2 + 64{,}000s + 16 \times 10^8)}.$$

$$i_L = (15 \sin 40{,}000t - 25e^{-32{,}000t} \sin 24{,}000t)u(t) \text{ mA}.$$

Practice Examples

Questions?? Thoughts??



EE 202
Electric Circuit Analysis

with

Dr. Naveed R. Butt

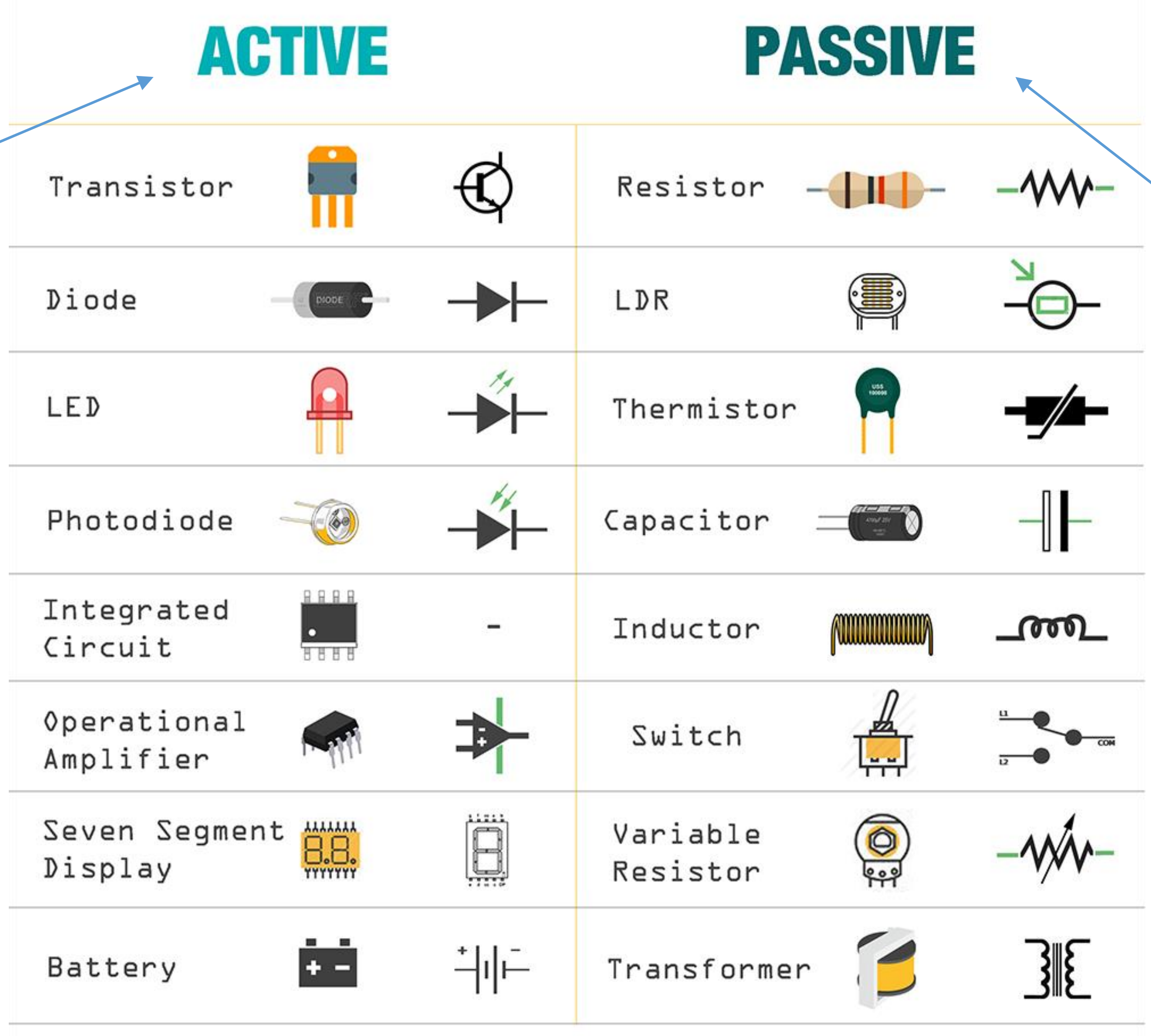
@

Jouf University

Elements: Active vs Passive

Provide (net) energy to the circuit

Require external power source to operate



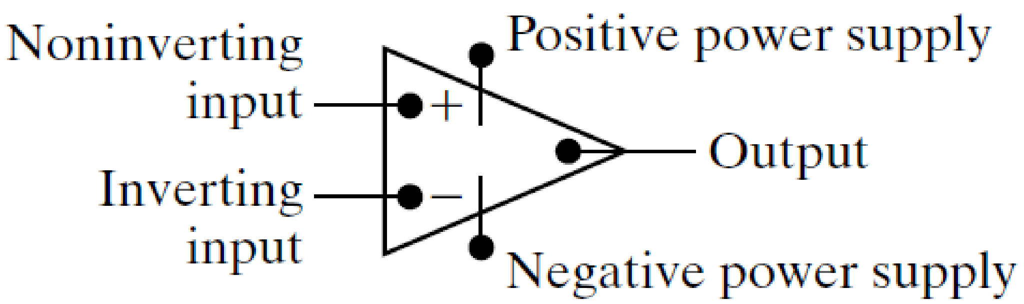
Do not provide (net) energy to the circuit

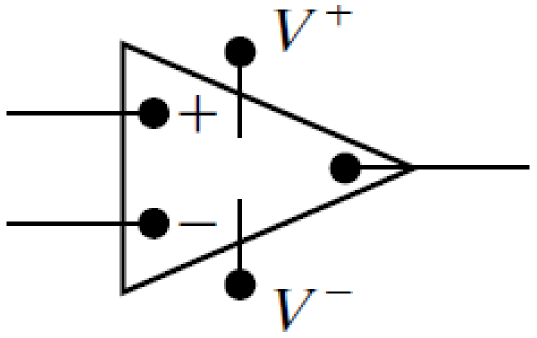
Sensors: some interesting problems …

Operational Amplifiers (Op-Amps)

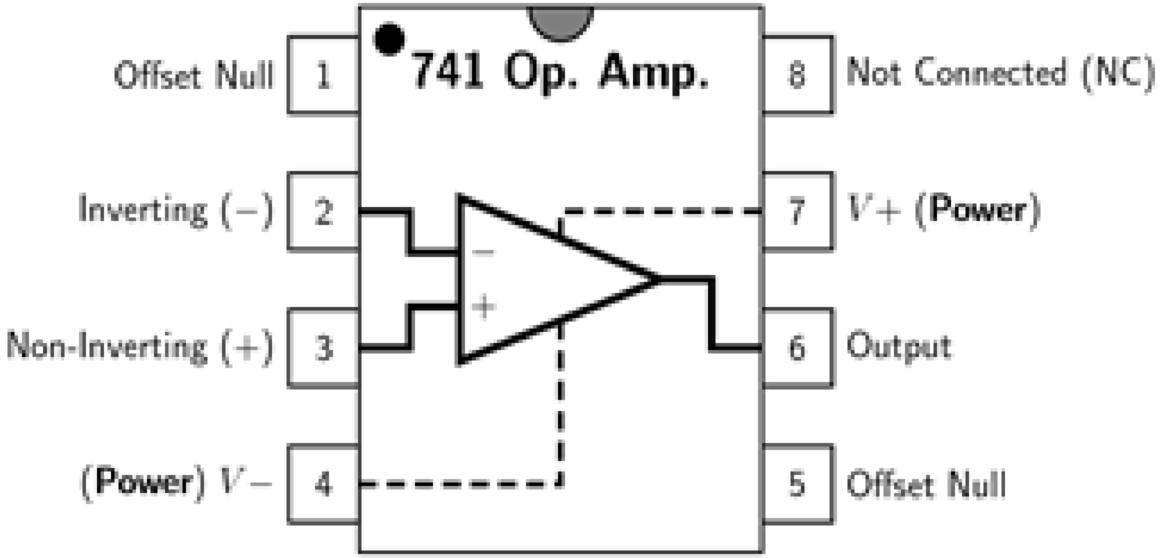
- An electronic component
- An active component
  - i.e. requires external power to operate and provides net energy to circuit
- In its very basic form: **a voltage “amplifying” device**
  - We can also call it “a voltage-controlled voltage source”
- Can perform several useful operations
  - Examples: voltage amplification, addition, subtraction, integration etc.
- Becomes “operational” when:
  - We connect different elements (resistors, capacitors etc.) to its terminals
  - Configuration of these external elements decides which operation it performs

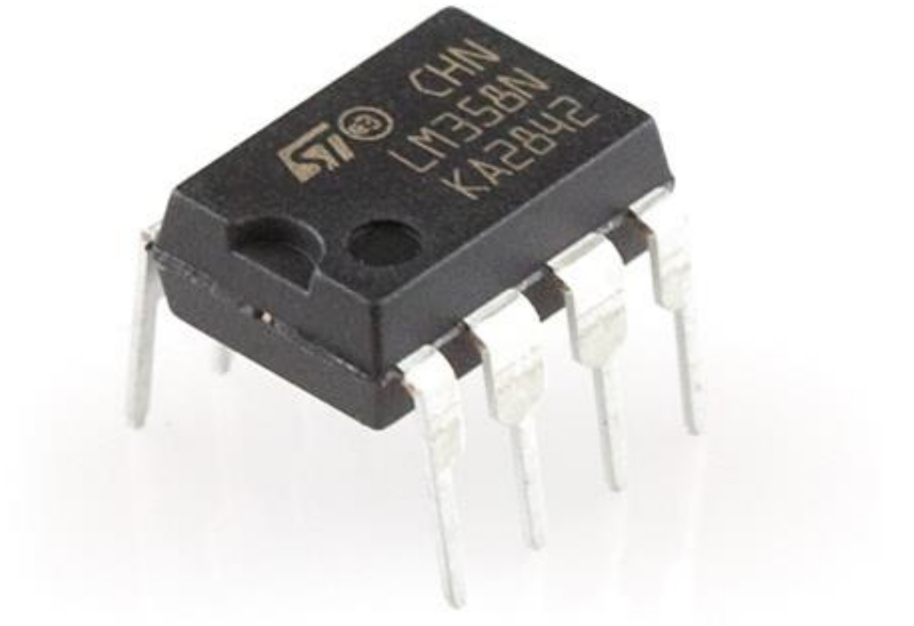
Op-Amp: terminals and symbol



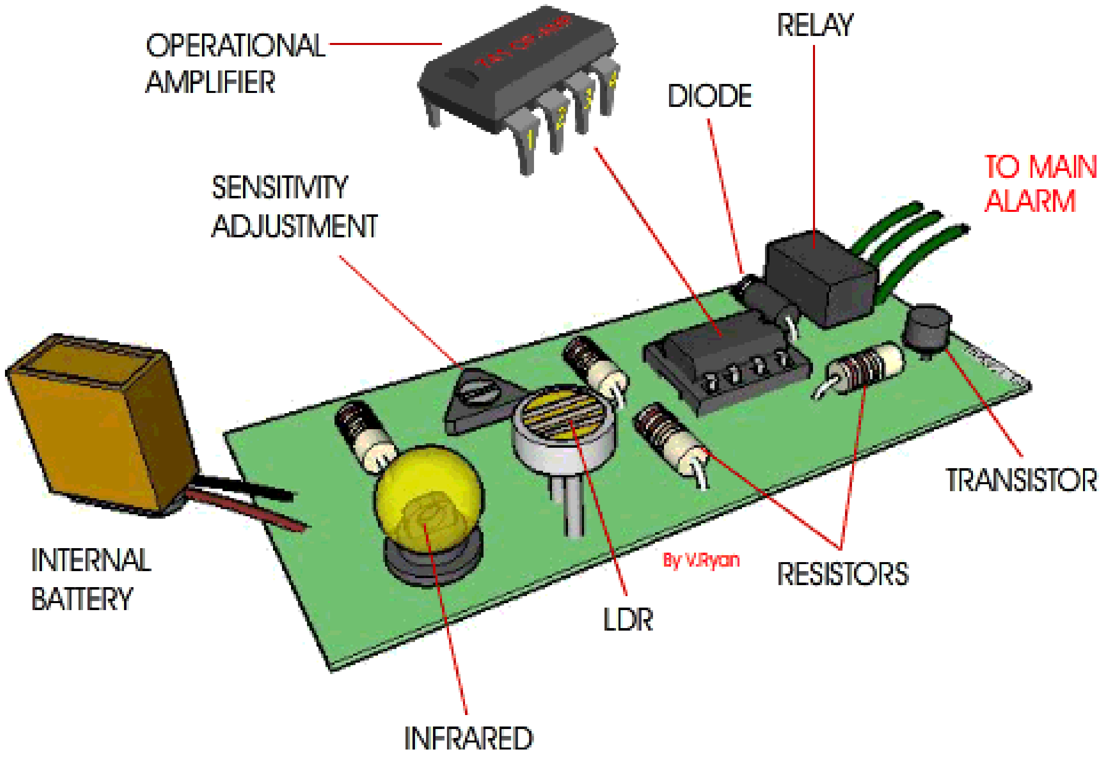


Op-Amp: in a package

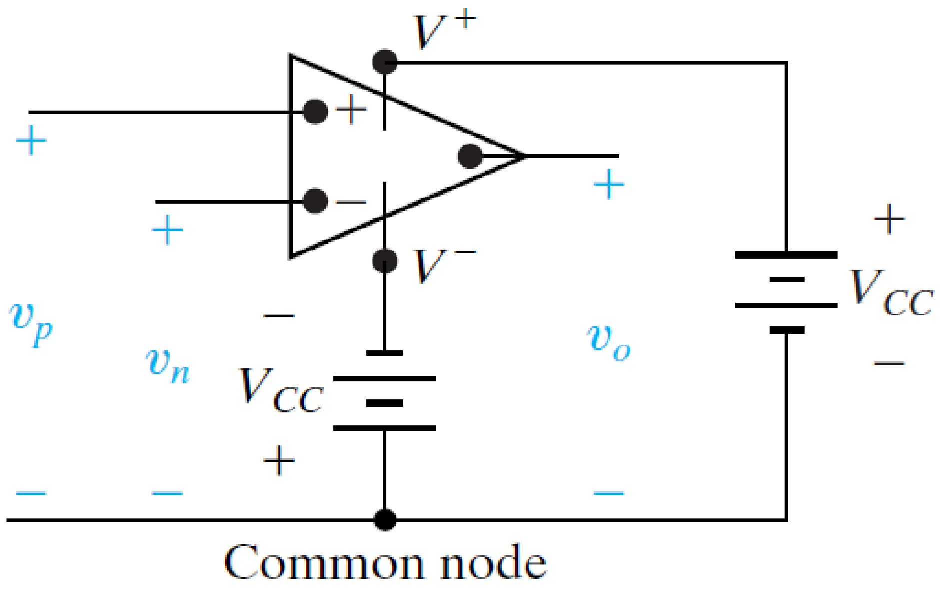


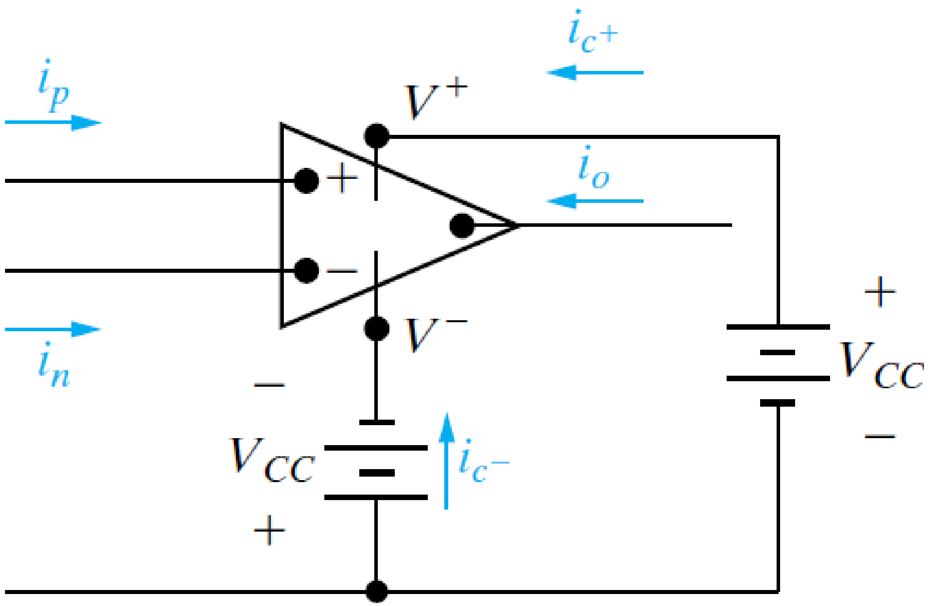


Op-Amp: in a circuit



Terminal Variables: voltages and currents





Ideal Op-Amp Characteristics

1. Infinite input impedance

$$i_p = i_n = 0.$$

Zero input currents

$$v_p = v_n.$$

Zero offset-voltage ($v_p - v_n$)

2. Zero output impedance

3. Infinite gain (when open-loop)

Open-loop: without any feedback
Gain: scale of amplification

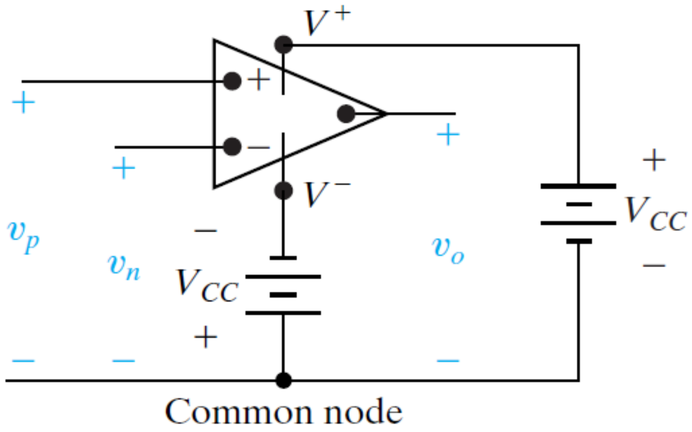
4. Infinite bandwidth

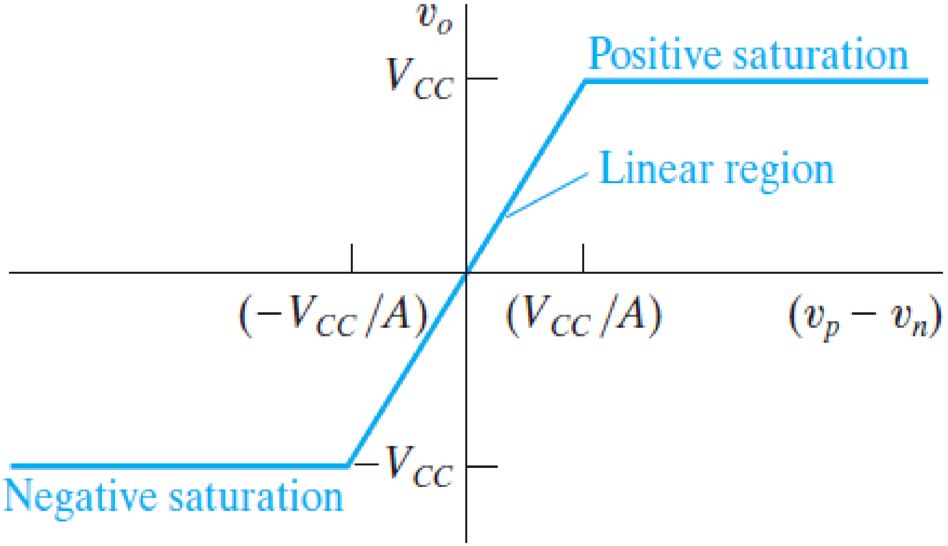
Bandwidth: range of frequencies for which the op-amp works well

Op-Amp: Ideal vs Realistic

	Ideal	Realistic	Typical Range
Input Impedance	∞	Extremely High	$10^5 - 10^{13}\Omega$
Output Impedance	0	Extremely Low	$10 - 100\ \Omega$
Open-loop Gain (A)	∞	Extremely High	$10^5 - 10^8$
Bandwidth	∞	Works well for all frequencies within range of operation	Varies by application

Op-Amp: Operating Ranges

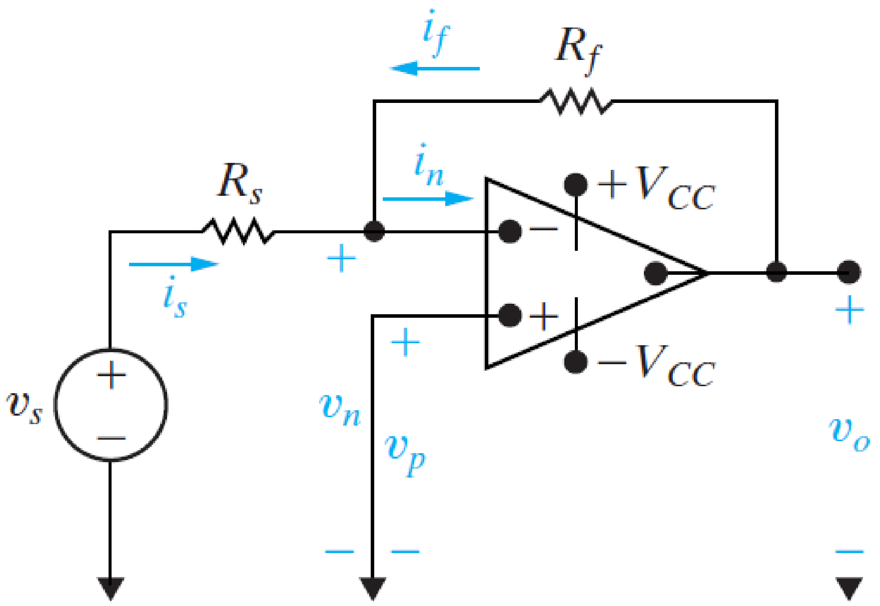




$A = gain$

$$v_o = \begin{cases} -V_{CC} & A(v_p - v_n) < -V_{CC}, \\ A(v_p - v_n) & -V_{CC} \le A(v_p - v_n) \le +V_{CC}, \\ +V_{CC} & A(v_p - v_n) > +V_{CC}. \end{cases}$$

Operation 1: Inverting Amplifier



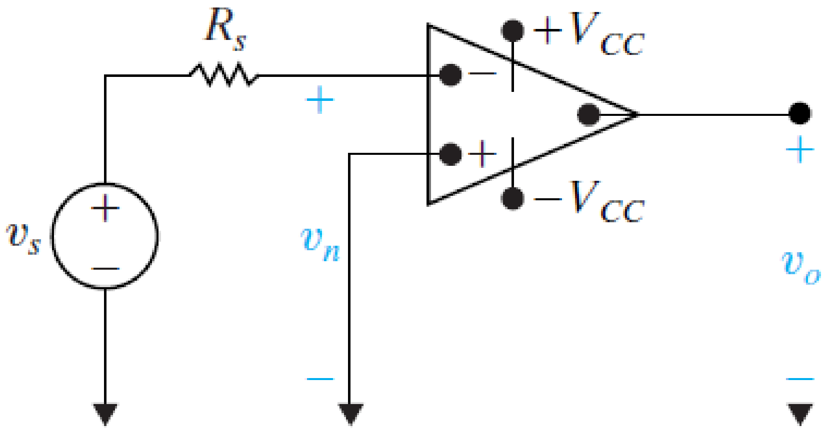
$$v_o = \frac{-R_f}{R_s} v_s .$$

$$Voltage\ Gain = A_v = \frac{v_o}{v_s} = -\frac{R_f}{R_s}$$

Upper limits →

$$|v_o| \leq V_{CC}, \qquad \left|\frac{R_f}{R_s} v_s\right| \leq V_{CC}, \qquad \frac{R_f}{R_s} \leq \left|\frac{V_{CC}}{v_s}\right| .$$

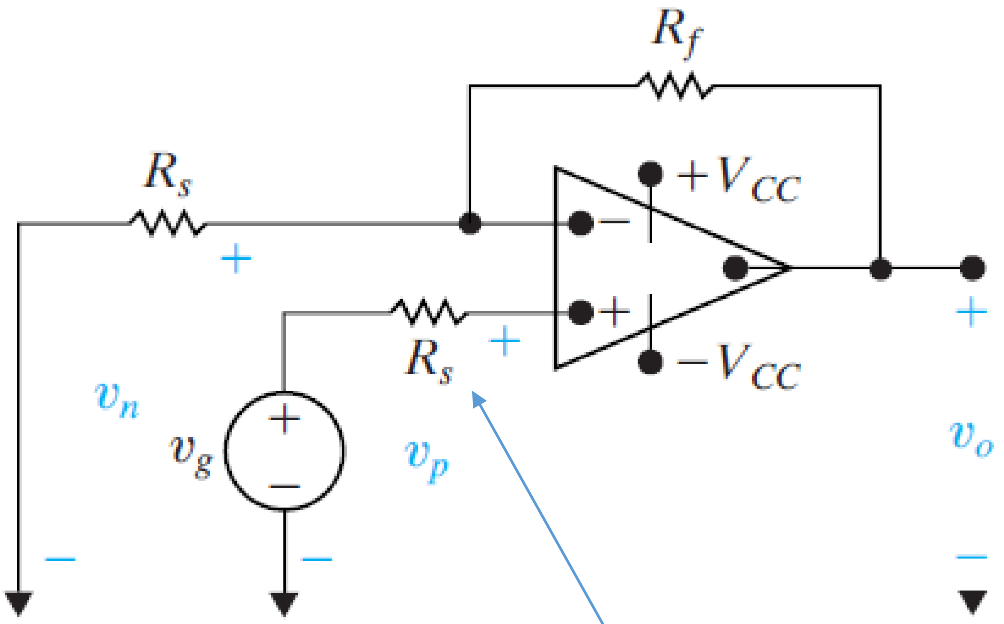
Inverting Amplifier in Open-Loop Setting



$$v_o = -Av_n,$$

Open-loop gain

Operation 2: Non-Inverting Amplifier

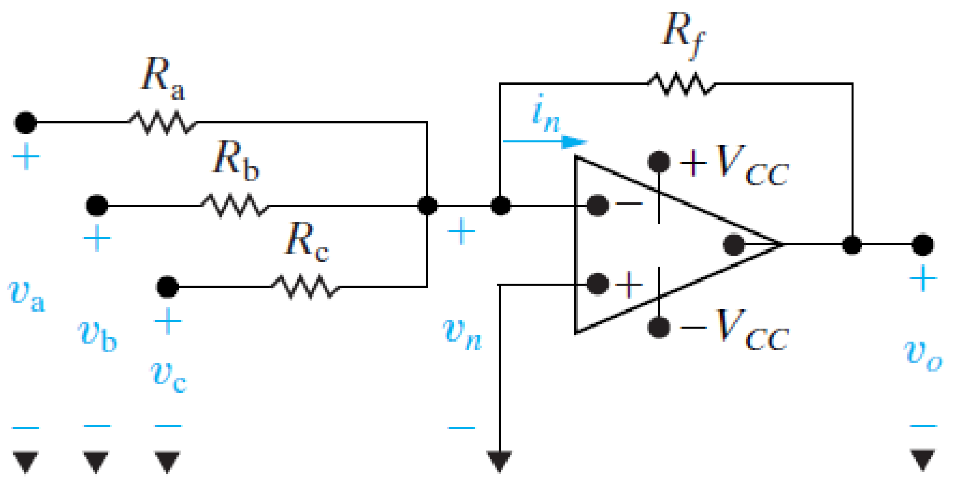


$$v_o = \frac{R_s + R_f}{R_s} v_g .$$

Results also hold if this resistor not there.

$$Voltage\ Gain = A_v = \frac{v_o}{v_g} = 1 + \frac{R_f}{R_s}$$

Operation 3: Summing Amplifier



$$v_o = -\left(\frac{R_f}{R_a}v_a + \frac{R_f}{R_b}v_b + \frac{R_f}{R_c}v_c\right).$$

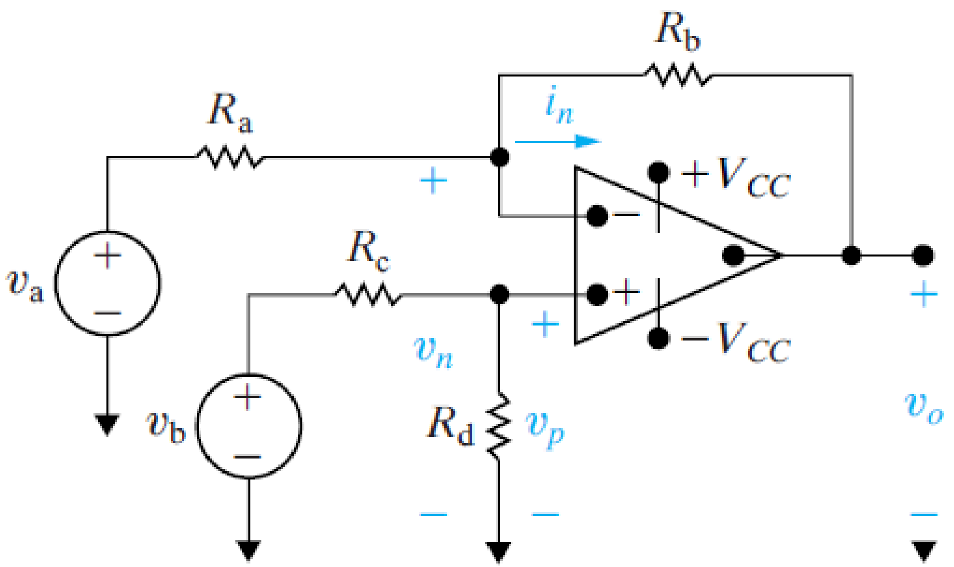
$$v_o = -\frac{R_f}{R_s}(v_a + v_b + v_c).$$

If $R_a = R_b = R_c = R_s$

$$v_o = -(v_a + v_b + v_c).$$

$R_f = R_s$

Operation 4: Difference Amplifier

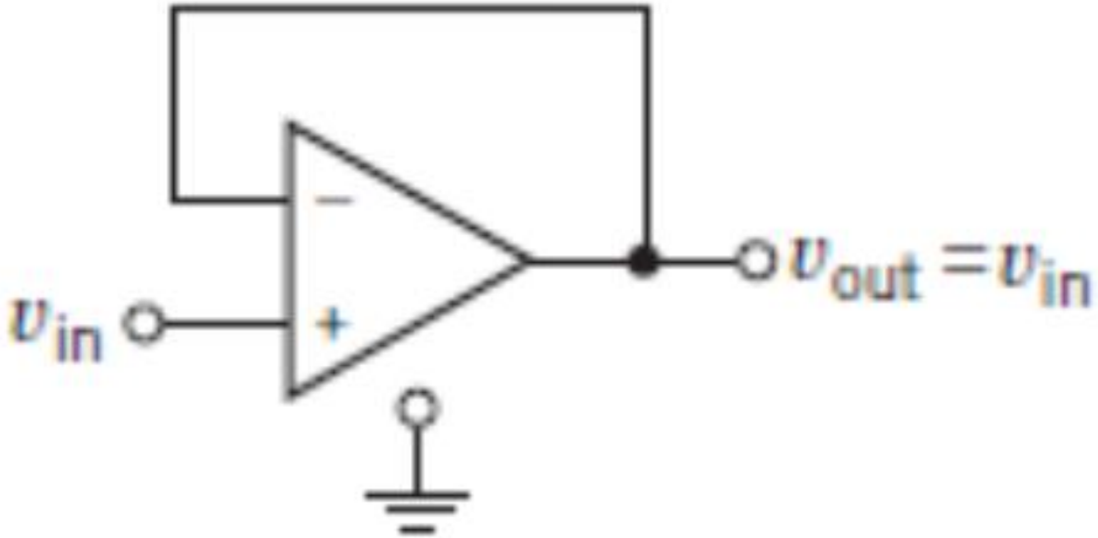


$$v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} v_b - \frac{R_b}{R_a} v_a.$$

$$v_o = v_b - v_a$$

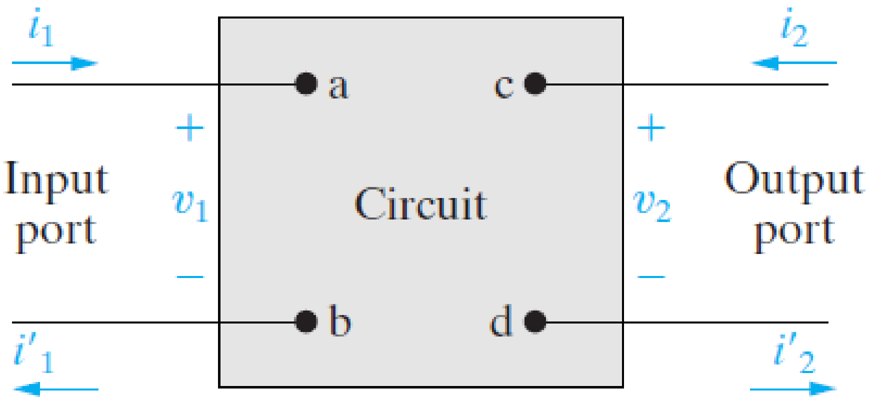
$$if \quad \frac{R_b}{R_a} = \frac{R_d}{R_c} = 1$$

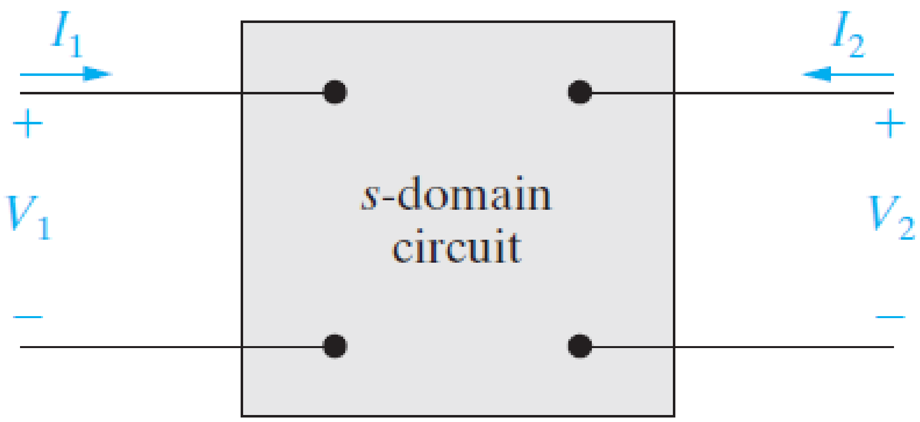
Operation 5: Voltage Follower (Buffer)



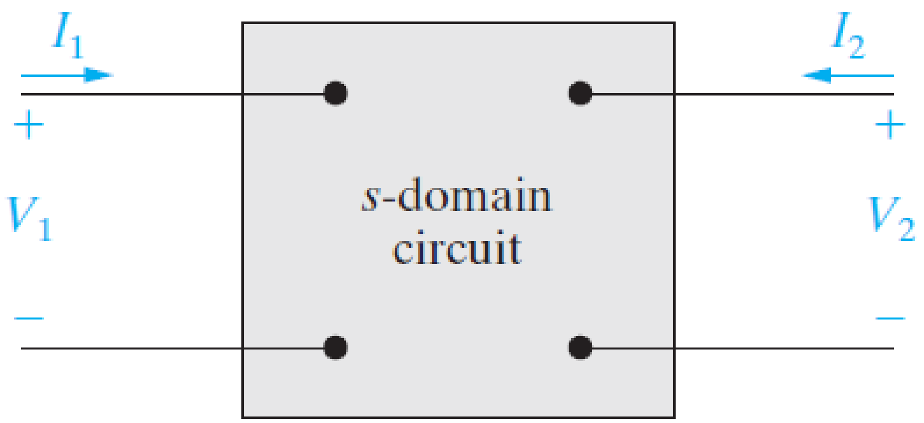
Practice Examples

Two-Port Networks





Two-Port Networks



$$V_1 = z_{11}I_1 + z_{12}I_2,$$
$$V_2 = z_{21}I_1 + z_{22}I_2;$$

Two-Port Networks

$$V_1 = z_{11} I_1 + z_{12} I_2,$$
$$V_2 = z_{21} I_1 + z_{22} I_2;$$

$$\Rightarrow$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Omega,$$
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega,$$
$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Omega,$$
$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega.$$

Practice Examples

Find the z parameters for the circuit shown in Fig. 18.3.

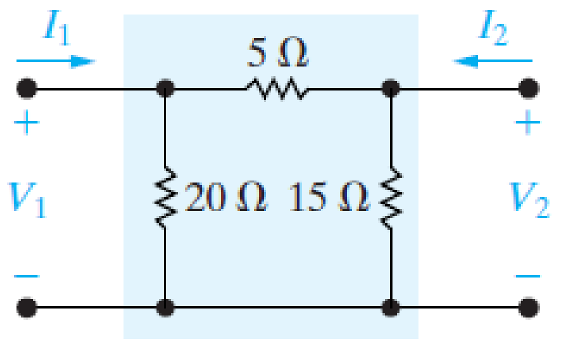


Figure 18.3 ▲ The circuit for Example 18.1.

Solution

The circuit is purely resistive, so the s-domain circuit is also purely resistive. With port 2 open, that is, $I_2 = 0$, the resistance seen looking into port 1 is the 20 Ω resistor in parallel with the series combination of the 5 and 15 Ω resistors. Therefore

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{(20)(20)}{40} = 10\ \Omega.$$

When I_2 is zero, V_2 is

$$V_2 = \frac{V_1}{15+5}(15) = 0.75V_1,$$

and therefore

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{0.75V_1}{V_1/10} = 7.5\ \Omega.$$

When I_1 is zero, the resistance seen looking into port 2 is the 15 Ω resistor in parallel with the series combination of the 5 and 20 Ω resistors. Therefore

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{(15)(25)}{40} = 9.375\ \Omega.$$

When port 1 is open, I_1 is zero and the voltage V_1 is

$$V_1 = \frac{V_2}{5+20}(20) = 0.8V_2.$$

With port 1 open, the current into port 2 is

$$I_2 = \frac{V_2}{9.375}.$$

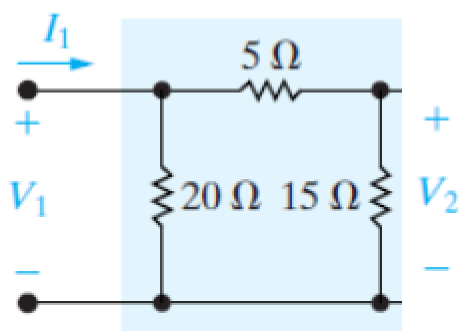
Hence

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{0.8V_2}{V_2/9.375} = 7.5\ \Omega.$$

Step 1: Solve for $I_2 = 0$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$z_{11} = 20 || (15+5) = 20||20 = \frac{20 \times 20}{40} = 10\Omega$$



$I_2 = 0,$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$\frac{V_2}{V_1} = \frac{15}{15+5}$$

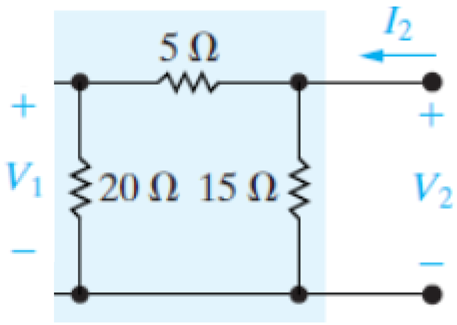
$$V_2 = \frac{15V_1}{20} = 0.75V_1$$

$$I_1 = \frac{V_1}{z_{11}} = 0.1V_1$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{0.75V_1}{V_1/10} = 7.5\ \Omega.$$

Step 2: Solve for $I_1 = 0$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



$$z_{22} = 15||(20+5) = 15||25 = \frac{15 \times 25}{40} = 9.375\Omega$$

$$\frac{V_1}{V_2} = \frac{20}{20+5} \qquad V_1 = 0.8V_2 \qquad I_2 = \frac{V_2}{z_{22}} = \frac{V_2}{9.375}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{0.8V_2}{V_2/9.375} = 7.5\ \Omega.$$

Step 3: Combine the results

$$V_1 = z_{11}I_1 + z_{12}I_2,$$

$$V_2 = z_{21}I_1 + z_{22}I_2;$$

$$V_1 = 10I_1 + 7.5I_2$$
$$V_2 = 7.5I_1 + 9.375I_2$$

Questions?? Thoughts??